## Cognitive Abilities Test"' Practice Activities Teacher Guide


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## Test 4: Number Analogies, Level 9

## Part 1: Overview of Number Analogies

An analogy draws parallels between objects or ideas. Successful learners habitually reason by analogy. Good analogies allow students to use what they already know when they are trying to understand or remember new ideas. Reasoning by analogy requires attending carefully to the ways in which two things are similar. Then this relationship is mapped onto something new. Performance on this test predicts mathematics achievement because discovering quantitative patterns and relationships is at the core of learning mathematics.

Each question on the Number Analogies test presents two pairs of numbers. To solve the questions, students must find a rule that transforms the first number in each pair into the second number. The same rule must work for both pairs of numbers. Then the student uses the rule to generate the missing number in the third pair. Most questions require only one rule, but some questions use two rules such as adding and then doubling. Students are allowed to use scratch paper on the actual test. Items in the test look like this:

## S1



What rule changes 12 to 4 and 16 to 8 ?

S2

$[12 \rightarrow 4] \quad[16 \rightarrow 8]$

J 16
K 12

L 10
M 8
N 6

When practicing the Number Analogies questions, encourage students to use these strategies.

- Think of a rule that describes how to transform the first number into the second number in the first two pairs. For example, the second number is 8 less than the first.
- If more than one rule works, then try the simplest rule first.
- Apply the same rule to the third pair of numbers to determine the missing number.

Students at this level tend to make the following common mistakes.

- Students may use the wrong arithmetic operation. For example, the student might add instead of subtract.
- Students might find a rule that works with only one of the pairs and not both. For example, in the first sample question, a student may remember that adding 7 to 7 will make 14. Although this rule does not work for the second pair, the student may simply add 7 to 1 , and mark the answer as 8 .


## Part 2: Number Analogies Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in blue italics: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how to do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.
(Make sure each student has a practice booklet. Then SAY:)
Open your practice booklet to page 1.
(Check that all students have the correct page.)

## P1

Let's do the first practice question. Look at the first two rows of the question.



A B C

10
D
(Hold up your copy of the booklet. Point to the first two rows as you SAY:)
Each row of the question has two numbers and an arrow between them. The arrows mean that the numbers 9 and 10 follow a rule, and the numbers 8 and 9 follow the same rule.

How do we go from 9 to 10?
(Encourage responses.)
How do we go from 8 to 9 ?
(Encourage responses.)

In this question, the rule for each pair is to add 1 to the first number to get the second number.
Now look at the third row. We must use the same rule for 7. What is the missing number?
(Encourage responses.)
The rule is to add 1 to the first number. 7 plus 1 is 8 . So the missing number is 8 which is answer choice B.
(Check to make sure that all students have selected answer choice B.)

## P2

Let's do the second practice question.

(Point to the first two rows of the question as you SAY:)
Look at the top two rows of the question.
In the first row, the numbers 10 and 8 follow a rule. The numbers 8 and 6 in the second row must follow the same rule.

How do we go from 10 to 8, and from 8 to 6?
(Encourage responses.)
The rule for this question is to subtract 2 from the first number to get the second number.
Now look at the third row. We must follow the same rule for 6 . What is the missing number?
(Encourage responses.)
When we subtract 2 from 6 we get 4 . So the missing number is 4 , which is answer choice $J$.
(Check to make sure that all students have selected answer choice J.)

## P3

Let's do the next practice question.


2

A

3

B

8

C

10

D
(Point to the first two rows of the question as you SAY:)
Look at the top two rows of the question. What rule do the first two pairs of numbers follow?
(Encourage responses. If a student suggests "add 7 to the first number" as the rule, SAY:)
The rule "add 7 to the first number" works for the first pair of numbers. But does it work for the next pair?
(Encourage responses.)
No, it doesn't. If we add 7 to 9 we get 16, not 18. So this rule does not work.
Is there another rule we can use?
(Encourage responses.)
In both pairs, when we double the first number we get the second number. When we double 7 we get 14 and when we double 9 we get 18 .

Using this rule, what is the missing number in the third pair?
(Encourage responses.)
When we double 1 we get 2. So the missing number is 2 which is answer choice A.
(Check to make sure that all students have selected answer choice A.)

Turn to the next page.

## P4

Let's do the next practice question.
$4 \quad[12 \rightarrow 4] \quad[16 \rightarrow 8] \quad[20 \rightarrow$ ? $]$
$\begin{array}{llllllllll}\text { J } & 16 & \text { K } & 12 & \text { L } & 10 & \text { M } & 8 & \text { N } & 6\end{array}$

Look at the numbers in the top row. This is the same kind of question as the questions we just answered, but now the number pairs are written in a row instead of in boxes. The arrows still mean that the two numbers must follow a rule.

What rule do the first two pairs of numbers follow? How do we go from 12 to 4 and from 16 to 8 ?
(Encourage responses.)
In this question, the rule for each pair is to subtract 8 from the first number to get the second number. When we subtract 8 from 12 we get 4 and when we subtract 8 from 16 we get 8 .

Now look at the third pair. We must follow the same rule. What is the missing number?

## (Encourage responses.)

The rule is to subtract 8 from the first number. 20 minus 8 is 12 . So the missing number is 12 which is answer choice $K$.
(Check to make sure that all students have selected answer choice K.)

## P5

Now look at the next practice question. Try to solve this practice question on your own.

$$
5 \quad[18 \rightarrow 9] \quad[12 \rightarrow 6] \quad[24 \rightarrow \text { ? }]
$$

A 19
B 18
C 15
D 13
E 12
(Make sure students have enough time to solve the problem. Then SAY:)
What is the missing number in the third pair?
(Encourage responses.)
How did you solve this question?
(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

What rule do the first two pairs of numbers follow?
(Encourage responses.)
In both pairs, when we divide the first number by 2 (or halve it) we get the second number.
(Note that the rule could also be "the second number is half of the first number.")
In the first pair, when we divide 18 by 2 we get 9 . In the second pair, when we divide 12 by 2 we get 6 . Using this rule, what is the missing number in the third pair?
(Encourage responses.)
When we divide 24 by 2 we get 12. So the missing number is 12 which is answer choice $E$.
(Check to make sure that all students have selected answer choice E.)

## P6

Now let's do the last practice question.
$6 \quad[2 \rightarrow 5]$
[4 $\rightarrow$ 9]
[3 $\rightarrow$ ? $]$
$\begin{array}{llllllllll}\text { J } & 4 & \text { K } & 5 & \text { L } & 6 & \text { M } & 7 & \text { N } & 8\end{array}$

Look at the first two pairs of numbers. How do we go from 2 to 5 and from 4 to 9 ?
(Encourage responses.)
Let me give you a hint: The rule for each pair in this question requires two changes.
(Encourage students to discover the two rules.)

The rule for this question is to double the first number and then add 1 to get the second number. In the first pair, we double 2 to get 4 and then 4 plus 1 is 5 .

Let's see if this rule works for the second pair as well.
In the second pair, we double 4 to get 8 and then 8 plus 1 is 9 .
Now look at the third pair. We must follow the same rule with 3. What is the missing number?
(Encourage responses.)
When we double 3 we get 6 and then 6 plus 1 is 7 . The missing number is 7 which is answer choice $M$.
(Check to make sure that all students have selected answer choice M.)

## Test 5: Number Puzzles, Level 9

## Part 1: Overview of Number Puzzles

The Number Puzzles test requires the student to solve simple equations by choosing an answer that makes the numbers on both sides of the equal sign the same amount. To answer the questions in this test, the student must find the missing number that goes in the square with the question mark. This test measures understanding of mathematical identities.

In this test, students are asked to solve problems that look like this:

A 3
B 4
C 5
D 6
E 14

When practicing the Number Puzzles questions, encourage students to use these strategies.

- Perform the necessary calculations to make both sides of the equal sign total the same amount in each row of the question.
- Substitute numbers for shapes wherever possible. For example, in the above sample question, the second row shows that the diamond is equal to 5 . The number 5 can replace the diamond in the top row.
- Use scratch paper to keep track of each step in the solution.

Students at this level tend to make the following common mistakes.

- Students may work from the top down rather than from the bottom up. Solving the simple identities in the second and third lines is necessary before one can solve the identity in the first line.
- Students might use the wrong arithmetic operation. For example, the student may add numbers instead of subtracting them.
- Students may solve the question for a shape other than the square with the question mark. For example, in the sample question above, the student might solve for the diamond and choose 5.


## Part 2: Number Puzzles Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in blue italics: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.
(Make sure each student has a practice booklet. Then SAY:)
Open your practice booklet to page 3.
(Check that all students have the correct page.)

## P1

Let's do the first practice question together.
(Hold up your copy of the practice booklet. Point to the numbers and signs in the shaded rectangle as you SAY:)

A 4
B 5
C 6
D 10

Which number goes in the square with the question mark?
(Encourage responses.)
Both sides of the equal sign must total the same amount. The left side is 3 . What number must we subtract from 7 to make the right side of the equal sign also total 3 ?
(Encourage responses.)
To find the missing number we must subtract 4.7 minus 4 is 3 . Now the numbers on both sides of the equal sign total 3.
(Point to the answer choices as you SAY:)

The number 4, which is answer choice A, is the correct answer.
(Check to make sure that all students have selected answer choice A.)

## P2

Let's do the next practice question.

$$
\begin{aligned}
& 8=6+5-? ? \\
& \text { Ј } 11 \quad \text { K } 4
\end{aligned}
$$

Which number goes in the square with the question mark?
(Encourage responses.)
The left side of the equal sign is 8 . The right side of the equal sign shows 6 plus 5 , which is 11 . What number must we subtract from 11 to make both sides of the equal sign total 8 ?
(Encourage responses.)
We must subtract 3 because 11 minus 3 is 8 .
(Point to the answer choices as you SAY:)
The number 3, which is answer choice $L$, is the correct answer.
(Check to make sure that all students have selected answer choice L.)

## P3

Let's do the next practice question.

## 3 <br> ? $+\diamond=9$ $\diamond=5$

A 3
B 4
c 5
D 6
E 14

This question has two rows of numbers and shapes. In both rows, both sides of the equal sign must total the same amount.

Look at the top row of the question. The right side of the equal sign is 9. The left side shows the square with the question mark plus the diamond. So this side must total 9 to equal the right side of the equal sign.

But how much is the diamond?
(Pause.)
The second row shows that the diamond is equal to 5. If we replace the diamond in the first row with 5, then a number plus 5 must equal 9.
(Make sure that all students understand how to substitute the number 5 for the diamond. Provide another example if necessary.)

Which number goes in the square with the question mark?
(Encourage responses. Then point to the answer choices as you SAY:)
The correct answer is 4 because 4 plus 5 is 9 . Answer choice B is the correct answer.
(Check to make sure that all students have selected answer choice B. Encourage them to work from the bottom up rather than from the top down.)

## P4

Let's do the next practice question.

J 11 K 10
L 9
M 8
N 7

Should we start with the top row or with the second row?
We should start with the second row because it will tell us how much the diamond is.
(Point to the second row as you SAY:)
The second row shows that the diamond is equal to 1. If we replace the diamond in the first row with 1 , then 1 subtracted from the square with the question mark must equal 9.

Which number goes in the square with the question mark?
(Encourage responses. Then point to the answer choices as you SAY:)
The correct answer is 10 because 10 minus 1 is 9 . So answer choice $K$ is the correct answer.
(Check to make sure that all students have selected answer choice K.)
Turn to the next page.

## P5

Look at the next practice question. Try to solve this practice question on your own.

A 21
B 16
C 14
D 9
E 7
(Make sure students have enough time to solve the problem. Then SAY:)
Which number goes in the square with the question mark?
(Encourage responses.)

How did you solve this question?
(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

The second row shows that the diamond minus 8 is equal to 6. So the diamond must be 14 since 14 minus 8 is 6 .

If we replace the diamond in the first row with 14, then the square with the question mark plus 7 must equal 14.

Which number goes in the square with the question mark?
(Encourage responses. Then point to the answer choices as you SAY:)
The correct answer is 7 because 7 plus 7 is 14. Answer choice $E$ is the correct answer.
(Check to make sure that all students have selected answer choice E.)

## P6

Look at the last practice question. Try to solve this practice question on your own.

(Make sure students have enough time to solve the problem. Then SAY:)
Which number goes in the square with the question mark?
(Encourage responses.)
How did you solve this question?
(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

There are three rows in this question. In the top row, the square with the question mark equals the diamond plus 6 .

But how much is the diamond?
(Encourage responses. Then point to the second row as you SAY:)
In the second row, 12 equals the diamond minus the circle.
But how much is the circle?
(Encourage responses. Then point to the third row as you SAY:)
So we need to start with the bottom row to solve the problem.
The third row shows that the circle is the same as 5.
If we replace the circle in the second row with 5, then the diamond minus 5 must be equal to 12 . So, the diamond must be 17 since 17 minus 5 is 12 .

If we replace the diamond in the first row with 17, then the square with the question mark must be equal to 17 plus 6 .

Which number goes in the square with the question mark?
(Encourage responses. Then point to the answer choices as you SAY:)
The correct answer is 23, which is answer choice J, because 17 plus 6 is 23 .
(Check to make sure that all students have selected answer choice J.)

## Test 6: Number Series, Level 9

## Part 1: Overview of Number Series

At this level there are two Number Series question formats. The first type of question requires the student to infer the next number in a series of beads on an abacus. The second type of question presents the series in numerical form. These questions test the ability to see quantitative patterns and choose the simplest logical extension of the pattern from the choices available. These practice activities are designed to help students use and understand the logic and procedures necessary to solve the problems. Students will be allowed to use scratch paper in the actual test.

In this test, students are asked to solve problems that look like this:

## SI



## $435046 \rightarrow$

A 1
B 3
C 5
D 7
E 9


Which number should come next to continue the pattern?

When practicing the Number Series questions, encourage students to use these strategies.

- Think of (and say silently) a rule that describes the pattern in either the string of beads in the box or the series of numbers. For example, each string has one more bead than the previous one.
- Look for repeating patterns in groups of adjacent numbers. If this is not successful, then look for patterns in every other number or every third number. For example, every other number in the series 43546 increases by one.
- Apply the rule to determine which string of beads or which number will come next in the series.
- Eliminate answer choices that do not fit the rule.

Students at this level tend to make the following common mistakes.

- Students may choose an answer that is the same as a string of beads in the initial series of beads rather than choosing the answer that continues the series.
- Students might choose an answer based on its appearance and not the actual count of beads. For example, the student may recognize the increasing pattern in the first sample question above and select the fourth answer choice without counting the number of beads, simply because it has more beads than the strings in the box.
- Students may choose the answer choice that simply repeats the pattern. For example, in the first sample question above, the student might choose the first answer choice.
- Students might use the wrong arithmetic operation. For example, the second sample question above has a pattern that has two steps - first subtract 1 then add 2. The student might confuse these two steps and add instead of subtract.


## Part 2: Number Series Practice Test Script

The following script covers many issues that will help students do their best on the test. Read aloud the text printed in blue italics: these are directions to the students. Directions for you are in parentheses and should NOT be read aloud. Feel free to modify the script to ensure that students understand what they are supposed to do and how to do it.

It may be helpful to make copies of the practice questions in order to display them one at a time on an overhead projector. If this is not possible, hold up a copy of the student practice booklet and point to different parts of each practice question as you discuss them with the class.
(Make sure each student has a practice booklet. Then SAY:)
Open your test booklet to page 5.
(Check that all students have the correct page.)
In these questions, you must look carefully at the beads in the box. Then choose the string of beads that comes next in the pattern.

## P1

Let's do the first practice question together.
1


A
B
C
D
(Hold up your copy of the practice booklet and point to the beads in the box as you SAY:)
Look at the numbers below the strings of beads. Each number shows how many beads are on the string above it.

Now look at the beads in the box. What pattern do they make?
(Encourage responses.)
There are 4 beads, then 3 beads, then 2 beads, then 4, 3, and 2 beads again. What is the pattern?
(Encourage responses.)

The pattern is 4, 3, $2 \ldots 4,3,2$.
(Point to the empty space on the right side of the box as you SAY:)
How many beads should the next string have?
(Encourage responses.)
The next string should have 4 beads to repeat the pattern of 4 , then 3 , then 2 beads.
(Point to the answer choices as you SAY:)
Which string of beads in the answer choices shows this?
(Encourage responses.)
The last answer choice shows a string with 4 beads so answer choice D is correct.
(Check to make sure that all students have selected answer choice D.)

## P2

Let's do the next practice question.

(Point to the beads in the box as you SAY:)
Look at the beads in the box. What pattern do they make?
(Encourage responses. Explain that the horizontal line in the box and the answer choices separates the first five beads in a string from the remaining beads. It is there to help students count the longer strings of beads.)

There is 1 bead on the first string, 1 bead on the second string, 3 beads on the third string, 3 beads on the next string, then 5 beads, followed by another 5 beads. What is the pattern?

## (Encourage responses.)

The pattern is $1,1 \ldots 3,3 \ldots 5,5$.
(Point to the empty space on the right side of the box as you SAY:)
How many beads should the next string have?
(Encourage responses.)
The number of beads is getting larger, so the next string should have more than 5 beads. Which strings in the answer choices have more than 5 beads?

## (Encourage responses.)

The first answer choice has 7 beads and the second one has 6 beads. Which is the correct answer?
(Encourage responses.)
The string with 7 beads will follow the string with 5 beads because the pattern increases by 2 .
(Check to make sure that all students have selected answer choice J.)
In this question, you must first see that the number of beads is increasing, but then you must also count how many new beads are added at each step. The second answer choice cannot be the correct answer because it is increasing by only one bead, and not by two beads.

## P3

Look at the next practice question. Try to solve this practice question on your own.



A


B


C


D
(Make sure students have enough time to solve the problem. Then SAY:)
How many beads should the next string have?

## (Encourage responses.)

How did you solve this question?
(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

Let's count the beads on each string in the box. There is 1 bead, then 2 beads, then 4 beads, then 5 beads, then 7 beads, then 8 beads. What pattern do they make?

## (Encourage responses.)

Look at the beads on each string. How does each string differ from the one before it?
(Encourage responses.)
The number of beads gets larger by 1, then 2, then 1, then 2, and then 1 . So the pattern goes like this: add 1, add $2 \ldots$ add 1, add 2.
(Point to the empty space on the right side of the box as you SAY:)
How many beads should the next string have?

## (Encourage responses.)

The next step is to add 2 beads to the string with 8 beads. 8 plus 2 is 10 . So the string with 10 beads is the correct answer.
(Check to make sure that all students have selected answer choice D as you SAY:)
Let's look at this question again. Can you find any other way to solve it?
(Encourage responses.)
When you just look at every other string, how many beads are there on each string?
(Encourage responses. Then point to each string as you SAY:)
There is 1 bead on the first string, 4 beads on the third string, and 7 beads on the fifth string. The number of beads is increasing 1, 4, 7. The pattern increases by 3 on every other string.

How many beads should the last string have?
(Encourage responses.)
Since the pattern increases by 3 on every other string, the next step is to add 3 beads to the string with 7 beads. 7 plus 3 is 10. As you see, there can be different ways to find patterns and solve the problems.

Turn to the next page.

## P4

Let's answer the next practice question. This is the same kind of question, but now there are only numbers and no beads.
4
$6 \quad 10 \quad 14 \quad 18 \rightarrow$
$\begin{array}{lllllllll}\text { J } & 16 & \text { K } & 20 & \text { L } & 22 & \text { M } & 24 & \text { N }\end{array}$
(Point to the top row of numbers as you SAY:)
Look at the series of numbers in the top row. What pattern do the numbers in the series follow?
(Encourage responses.)
Each number is 4 more than the number before it. So the pattern is to add 4 to each number. Which number will come next in the series?
(Encourage responses.)
The pattern is to add 4 to each number. The last number is 18.18 plus 4 is 22 . So the next number in the series is 22 which is answer choice $L$.
(Check to make sure that all students have selected answer choice L.)

## P5

Let's do the next practice question.
$\begin{array}{lllllll}5 & 4 & 3 & 5 & 4 & 6 & \rightarrow\end{array}$
A 1
B 3
C 5
D 7
E 9
(Point to the top row of numbers as you SAY:)

What pattern do the numbers in the series follow?

## (Encourage responses.)

The first two numbers in the series are 4 and 3. To go from 4 to 3, we subtract 1. The next number is 5 . To go from 3 to 5 , we add 2. The next number is 4 . To go from 5 to 4 , we subtract 1 . The next number is 6 . To go from 4 to 6 , we add 2. The pattern is: subtract 1, add 2, subtract 1 , add 2, and so on.

Which number will come next in the series?
(Encourage responses.)
The next step is to subtract 1 from 6.6 minus 1 is 5 . So the next number in the series is 5 which is answer choice $C$.
(Check to make sure that all students have selected answer choice C.)
You can also solve this problem by looking for patterns in every other number. Every other number is increasing by 1 . The answer should be 5 because 4 plus 1 is 5 .

## P6

Look at the last practice question. Try to solve this practice question on your own.

(Make sure students have enough time to solve the problem. Then SAY:)
Which number will come next in the series?
(Encourage responses.)
How did you solve this question?
(Encourage responses. If there are students who don't understand how to solve the question, go through the process as follows.)

Look at every other number. Starting with the first number, what pattern does every other number follow?

## (Encourage responses.)

After the first number, every other number increases by 2. Starting with the second number, what pattern does every other number follow?
(Encourage responses.)
After the second number, every other number is 3.
(Point to the answer choices as you SAY:)
Which number will come next in the series?

## (Encourage responses.)

The next step is to add 2 to 7 . So the next number in the series will be 9 because 7 plus 2 is 9 . So answer choice $M$ is the correct answer.
(Check to make sure that all students have selected answer choice M.)

