Margie bought 3 apples at a cost of 50 cents per apple. She paid with a 5-dollar bill. How much change did Margie receive?

(A) $\$1.50$ **(B)** $\$2.00$ **(C)** $\$2.50$ **(D)** $\$3.00$ **(E)** $\$3.50$

2011 AMC 8, Problem #1—
"What was the total cost for three apples?"

Solution

Answer (E): The cost of the apples was $3 \times \$0.50 = \$1.50$. Her change was $\$5.00 - \$1.50 = \$3.50$.

Difficulty: Easy
SMP-CCSS: 1,4
CCSS-M: 5.OA1

Karl's rectangular vegetable garden is 20 feet by 45 feet, and Makenna's is 25 feet by 40 feet. Whose garden is larger in area?

(A) Karl's garden is larger by 100 square feet.
(B) Karl's garden is larger by 25 square feet.
(C) The gardens are the same size.
(D) Makenna's garden is larger by 25 square feet.
(E) Makenna's garden is larger by 100 square feet.

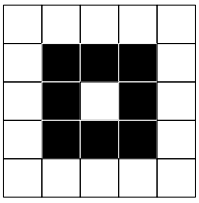
2011 AMC 8, Problem #2—
"Calculate areas for both gardens separately. "

Solution

Answer (E): Karl's garden is $20 \times 45 = 900$ square feet. Makenna's garden is $25 \times 40 = 1000$ square feet. Makenna's garden is larger by $1000 - 900 = 100$ square feet.

Difficulty: Medium Easy
SMP-CCSS: 1,5
CCSS-M: 7.G6

Extend the square pattern of 8 black and 17 white square tiles by attaching a border of black tiles around the square. What is the ratio of black tiles to white tiles in the extended pattern?

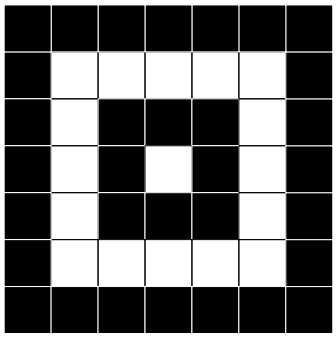


(A) $8:17$ **(B)** $25:49$ **(C)** $36:25$ **(D)** $32:17$ **(E)** $36:17$

2011 AMC 8, Problem #3—
"How many black tiles and white tiles are there in the extended pattern? "

Solution

Answer (D): There are 32 black tiles and 17 white tiles in the extended pattern. So the ratio is $32:17$.



Difficulty: Medium Easy
SMP-CCSS: 2, 7
CCSS-M: 6.RP1

Here is a list of the numbers of fish that Tyler caught in nine outings last summer:

$$2, 0, 1, 3, 0, 3, 3, 1, 2.$$

Which statement about the mean, mode, and median of these numbers is true?

(A) median $<$ mean $<$ mode **(B)** mean $<$ mode $<$ median
(C) mean $<$ median $<$ mode **(D)** median $<$ mode $<$ mean
(E) mode $<$ median $<$ mean

2011 AMC 8, Problem #4—
"What is the ordered list of the number of fish caught? "

Solution

Answer (C): The ordered list is 0, 0, 1, 1, 2, 2, 3, 3, 3. The mean is $\frac{15}{9} = \frac{5}{3}$, the median is 2, and the mode is 3. Because $\frac{5}{3} < 2 < 3$, the correct order is mean $<$ median $<$ mode.

Difficulty: Medium
SMP-CCSS: 2, 5
CCSS-M: 6.SP5C, 6.EE5

What time was it 2011 minutes after the beginning of January 1, 2011?

(A) January 1 at 9:31PM

(B) January 1 at 11:51PM

(C) January 2 at 3:11AM

(D) January 2 at 9:31AM

(E) January 2 at 6:01PM

2011 AMC 8, Problem #5—
"How many hours and minutes are equivalent to 2011 minutes? "

Solution

Answer (D): Convert 2011 minutes to 33 hours and 31 minutes. January 1 uses 24 hours. January 2 gets the remainder of the 9 hours and 31 minutes. The time at the end of 2011 minutes was 9:31AM on January 2.

Difficulty: Medium Easy
SMP-CCSS: 2
CCSS-M: 4.MD1

In a town of 351 adults, every adult owns a car, a motorcycle, or both. If 331 adults own cars, and 45 adults own motorcycles, how many of the car owners do not own a motorcycle?

(A) 20 **(B)** 25 **(C)** 45 **(D)** 306 **(E)** 351

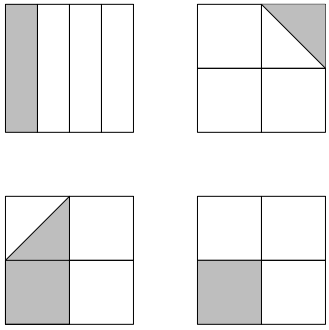
2011 AMC 8, Problem #6—
"Drawing a Venn diagram may help. "

Solution

Answer (D): In a population of 351 people, 45 people own a motorcycle. Therefore there are $351 - 45 = 306$ car owners who do not own a motorcycle.

Difficulty: Medium
SMP-CCSS: 1,2
CCSS-M: 7.EE3

Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially shaded. What percent of the total area is partially shaded?



(A) $12\frac{1}{2}$ **(B)** 20 **(C)** 25 **(D)** $33\frac{1}{3}$ **(E)** $37\frac{1}{2}$

2011 AMC 8, Problem #7—
"Find the shaded portion of each square separately. "

Solution

Answer (C): The upper left and the lower right squares are each one-fourth shaded, for a total of one-half square. The shaded portions of the upper right and lower left squares make up one-half square. So the total shaded area is one full square, which is 25% of the total area.

Difficulty: Medium
SMP-CCSS: 2, 7
CCSS-M: 6.G1, 6.RP3C

Bag A contains three chips labeled 1, 3, and 5. Bag B contains three chips labeled 2, 4, and 6. If one chip is drawn from each bag, how many different values are possible for the sum of the two numbers on the chips?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 9

2011 AMC 8, Problem #8— "Make a table of all possibilities. "

Solution

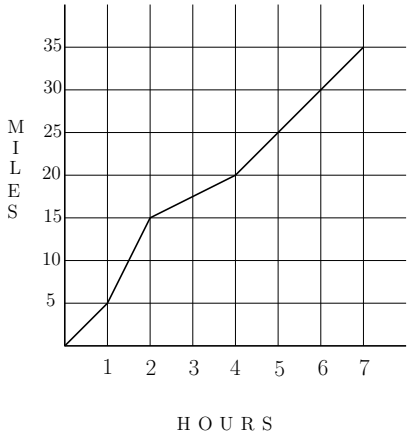
Answer (B): Make a table of possibilities.

+	1	3	5
2	3	5	7
4	5	7	9
6	7	9	11

The possible sums are 3, 5, 7, 9, and 11, for a total of 5 possibilities.

Difficulty: Medium Hard
SMP-CCSS: 8
CCSS-M: 7.SP8

Carmen takes a long bike ride on a hilly highway. The graph indicates the miles traveled during the time of her ride. What is Carmen's average speed for her entire ride in miles per hour?



(A) 2 **(B)** 2.5 **(C)** 4 **(D)** 4.5 **(E)** 5

2011 AMC 8, Problem #9—
"Find the total distance and hours that Carmen traveled. "

Solution

Answer (E): Carmen covers 35 miles in 7 hours, making her average speed $\frac{35}{7} = 5$ mph.

Difficulty: Medium Easy
SMP-CCSS: 2
CCSS-M: 6.RP3

The taxi fare in Gotham City is $\$2.40$ for the first $\frac{1}{2}$ mile and additional mileage charged at the rate $\$0.20$ for each additional 0.1 mile. You plan to give the driver a \$2 tip. How many miles can you ride for \$10 ?

(A) 3.0 **(B)** 3.25 **(C)** 3.3 **(D)** 3.5 **(E)** 3.75

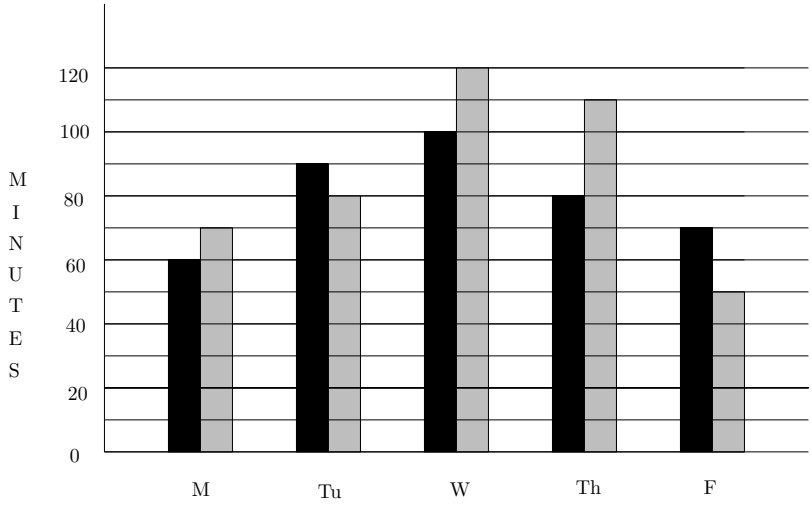
2011 AMC 8, Problem #10—
"What is the cost for the ride after the first 0.5 mile? "

Solution

Answer (C): Including a \$2 tip, a 0.5 mile ride would cost $\$4.40$. The remaining $\$5.60$ would take you an additional $0.1 \times \frac{5.60}{0.2} = 2.80$ miles, so the total distance is $0.5 + 2.8 = 3.3$ miles.

Difficulty: Medium
SMP-CCSS: 4
CCSS-M: 7.EE3

The graph below shows the number of minutes studied by both Asha (black bar) and Sasha (grey bar) in one week. On the average, how many more minutes per day did Sasha study than Asha?



(A) 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 12

2011 AMC 8, Problem #11—
"What is Asha's total study time? "

Solution

Answer (A): Asha's study time totals $60 + 90 + 100 + 80 + 70 = 400$ minutes, for an average of $\frac{400}{5} = 80$ minutes per day. Sasha's total is $70 + 80 + 120 + 110 + 50 = 430$ minutes, for an average of $\frac{430}{5} = 86$ minutes per day, so Sasha averages 6 minutes more per day than Asha.

OR

The daily differences between Sasha and Asha are $+10$, -10, $+20$, $+30$, and -20 minutes for a total of $+30$ minutes. The average difference is $\frac{30}{5} = 6$ minutes per day.

Difficulty: Medium
SMP-CCSS: 4
CCSS-M: 6.SP4

Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the probability that Angie and Carlos are seated opposite each other?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

2011 AMC 8, Problem #12—
"Find all possibilities first. "

Solution

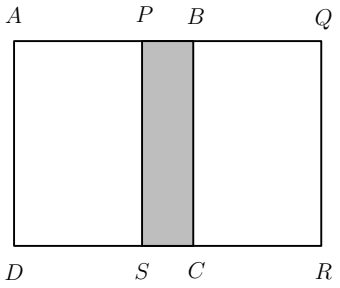
Answer (B): Proceeding clockwise from Angie, the seating could be: BCD, BDC, CBD, CDB, DBC, or DCB. In 2 of these 6 possibilities Carlos is opposite Angie, so the probability is $\frac{2}{6} = \frac{1}{3}$.

OR

If Angie sits down first, there are three equally likely places for Carlos to sit. Only one of these is opposite Angie. Thus the probability is $\frac{1}{3}$.

Difficulty: Medium Hard
SMP-CCSS: 2, 3
CCSS-M: 7.SP8C

Two congruent squares, $ABCD$ and $PQRS$, have side length 15. They overlap to form the 15 by 25 rectangle $AQRD$ shown. What percent of the area of rectangle $AQRD$ is shaded?



(A) 15 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 25

2011 AMC 8, Problem #13—
"What are the shaded rectangle's height and length? "

Solution

Answer (C): The shaded rectangle $PBCS$ has height $BC = 15$ and length $SC = DC + SR - DR = 15 + 15 - 25 = 5$.
Rectangle $AQRD$ has the same height and length 25. The portion of rectangle $AQRD$ that is shaded is $\frac{15\times5}{15\times25} = \frac{5}{25}$, which is 20%.

Difficulty: Medium
SMP-CCSS: 2,5
CCSS-M:7.G1, 8.G2

There are 270 students at Colfax Middle School, where the ratio of boys to girls is $5 : 4$. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is $4 : 5$. The two schools hold a dance and all students from both schools attend. What fraction of the students at the dance are girls?

(A) $\frac{7}{18}$ **(B)** $\frac{7}{15}$ **(C)** $\frac{22}{45}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{23}{45}$

2011 AMC 8, Problem #14—
"What is the number of girls at the dance? "

Solution

Answer (C): The number of girls at the dance is $\frac{4}{9}(270) + \frac{5}{9}(180) = 120 + 100 = 220$. So the fraction of the students that are girls is $\frac{220}{450} = \frac{22}{45}$.

Difficulty: Medium Hard
SMP-CCSS: 2, 4
CCSS-M: 7.SP1

How many digits are in the product $4^5 \cdot 5^{10}$?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 15

2011 AMC 8, Problem #15—
"Rewrite 4^5 in another format."

Solution

Answer (D): The product $4^5 \cdot 5^{10} = 2^{10} \cdot 5^{10} = 10^{10}$ is a number with a 1 followed by 10 zeros for a total of 11 digits.

Difficulty: Medium Hard
SMP-CCSS: 5, 7
CCSS-M:8.EE1

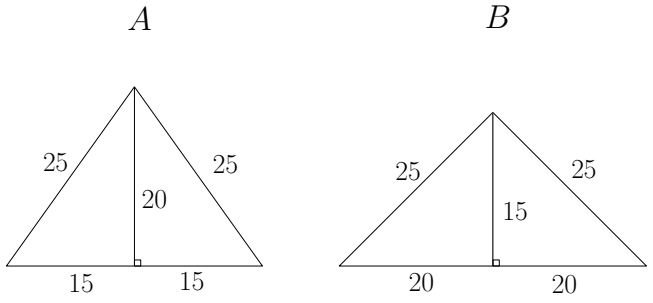
Let A be the area of a triangle with sides of length 25, 25, and 30. Let B be the area of a triangle with sides of length 25, 25, and 40. What is the relationship between A and B?

(A) $A = \frac{9}{16}B$ **(B)** $A = \frac{3}{4}B$ **(C)** $A = B$ **(D)** $A = \frac{4}{3}B$ **(E)** $A = \frac{16}{9}B$

2011 AMC 8, Problem #16—
"What is the height of each triangle?"

Solution

Answer (C):



The altitude shown divides each triangle into two congruent right triangles. The hypotenuse of each right triangle is 25. In $\triangle A$ the horizontal leg of each right triangle is 15, so the vertical leg is $\sqrt{25^2 - 15^2} = 20$. In $\triangle B$ the horizontal leg of each right triangle is 20, so the vertical leg is 15. The area of $\triangle A$ is $\frac{1}{2}(30)(20) = 300$, and the area of $\triangle B$ is $\frac{1}{2}(40)(15) = 300$, so the two areas are equal.

Difficulty: Hard
SMP-CCSS: 1,5
CCSS-M: 7.G6, 7.RP2B, 8.G7

Let w, x, y, and z be whole numbers. If $2^w \cdot 3^x \cdot 5^y \cdot 7^z = 588$, then what does $2w + 3x + 5y + 7z$ equal?

(A) 21 **(B)** 25 **(C)** 27 **(D)** 35 **(E)** 56

2011 AMC 8, Problem #17—
"Factor 588 first. "

Solution

Answer (A): Factor 588 into $2^2 \cdot 3^1 \cdot 5^0 \cdot 7^2$. Thus $w = 2, x = 1, y = 0$, and $z = 2$, and $2w+3x+5y+7z = 21$.

Difficulty: Hard
SMP-CCSS: 1, 2
CCSS-M: 8.EE1

A fair six-sided die is rolled twice. What is the probability that the first number that comes up is greater than or equal to the second number?

(A) $\frac{1}{6}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{5}{6}$

2011 AMC 8, Problem #18—
"Make a table of 36 possible equally-likely outcomes. "

Solution

Answer (D): Make a table of 36 possible equally-likely outcomes. The first number is greater than or equal to the second in the 21 cases indicated by the asterisks, so the probability is $\frac{21}{36} = \frac{7}{12}$.

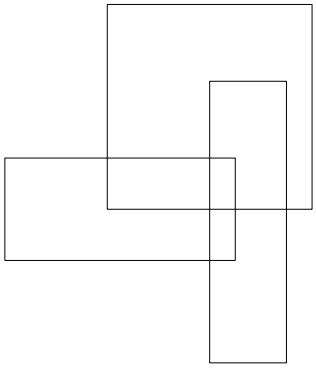
	1	2	3	4	5	6
1	$1, 1*$	$1, 2$	$1, 3$	$1, 4$	$1, 5$	$1, 6$
2	$2, 1*$	$2, 2*$	$2, 3$	$2, 4$	$2, 5$	$2, 6$
3	$3, 1*$	$3, 2*$	$3, 3*$	$3, 4$	$3, 5$	$3, 6$
4	$4, 1*$	$4, 2*$	$4, 3*$	$4, 4*$	$4, 5$	$4, 6$
5	$5, 1*$	$5, 2*$	$5, 3*$	$5, 4*$	$5, 5*$	$5, 6$
6	$6, 1*$	$6, 2*$	$6, 3*$	$6, 4*$	$6, 5*$	$6, 6*$

OR

In 6 of the 36 possible outcomes the two numbers are equal. The first number is greater than the second in half of the remaining 30 outcomes, so the first number is greater than or equal to the second in $6 + 15 = 21$ outcomes. The probability is $\frac{21}{36} = \frac{7}{12}$.

Difficulty: Medium Hard
SMP-CCSS: 4
CCSS-M: 7.SP8B

How many rectangles are in this figure?

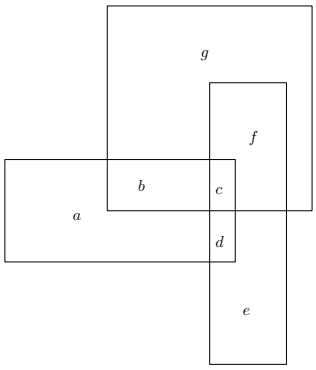


(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

2011 AMC 8, Problem #19—
"Partition the figure into non-overlapping regions. "

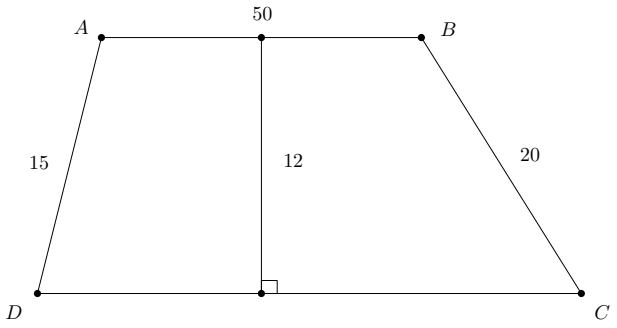
Solution

Answer (D): Partition the figure into non-overlapping regions as shown. The list of rectangles are b, c, d, ab, bc, cd, cf, de, $abcd$, $bcfg$, and $cdef$.



Difficulty: Hard
SMP-CCSS: 7
CCSS-M: 4.G2

Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 50$, $BC = 20$, and the altitude is 12. What is the area of the trapezoid?

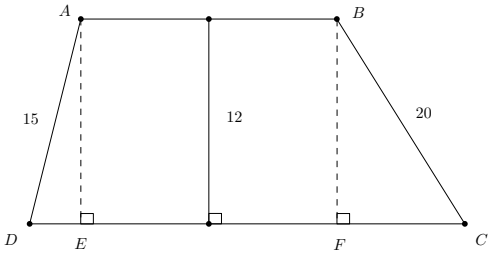


(A) 600 **(B)** 650 **(C)** 700 **(D)** 750 **(E)** 800

2011 AMC 8, Problem #20— "Adding appropriate perpendiculars would help. "

Solution

Answer (D): Let E and F be the feet of the perpendiculars from A and B to $\overline{DC}$. In right $\triangle AED$, $DE^2 = 15^2 - 12^2 = 225 - 144 = 81$, so $DE = 9$. In right $\triangle BFC$, $FC^2 = 20^2 - 12^2 = 400 - 144 = 256$, so $FC = 16$.



Right $\triangle AED$ has area $\frac{1}{2} \cdot 9 \cdot 12 = 54$, right $\triangle BFC$ has area $\frac{1}{2} \cdot 16 \cdot 12 = 96$, and rectangle $ABFE$ has area $50 \cdot 12 = 600$. The trapezoid $ABCD$ has area $54 + 96 + 600 = 750$.

OR

Begin as in the first solution and note that $DC = DE + EF + FC = 9 + 50 + 16 = 75$. Then the area of trapezoid is $\frac{1}{2}(AB + DC) \cdot AE = \frac{1}{2}(50 + 75) \cdot 12 = 125 \cdot 6 = 750$.

Difficulty: Medium Hard
SMP-CCSS: 5
CCSS-M: 7.G6

Students guess that Norb's age is 24, 28, 30, 32, 36, 38, 41, 44, 47, and 49. Norb says, "At least half of you guessed too low, two of you are off by one and my age is a prime number." How old is Norb?

(A) 29 **(B)** 31 **(C)** 37 **(D)** 43 **(E)** 48

2011 AMC 8, Problem #21— "How old is Norb at least? "

Solution

Answer (C): Because half are too low, Norb is at least 37. Because two are off by one, he must be between 36 and 38 or between 47 and 49. Because 37 is prime and 48 is not, Norb is 37.

Difficulty: Medium
SMP-CCSS: 1, 2
CCSS-M: None

What is the **tens** digit of 7^{2011} ?

(A) 0 **(B)** 1 **(C)** 3 **(D)** 4 **(E)** 7

2011 AMC 8, Problem #22—
"What are the last two digits of the previous power of 7? "

Solution

Answer (D): The tens digit of a power of 7 is determined by the last two digits of the previous power of 7. The pattern for the last two digits of successive powers of 7 is **0**1, **0**7, **4**9, **4**3, **0**1, **0**7, **4**9, **4**3, **0**1, **0**7, **4**9, **4**3, **0**1,.... Since $2011 = 4 \cdot 502 + 3$, the last two digits of 7^{2011} are 43 and the tens digit is 4.

Difficulty: Medium Hard
SMP-CCSS: 2, 7
CCSS-M: 8.EE1

How many 4-digit positive integers have four different digits, where the leading digit is not zero, the integer is a multiple of 5, and 5 is the largest digit?

(A) 24 **(B)** 48 **(C)** 60 **(D)** 84 **(E)** 108

2011 AMC 8, Problem #23—
"What are the units digits for an integer that is a multiple of 5? "

Solution

Answer (D): Any integer that is a multiple of 5 must have a 0 or 5 as the units digit.
If the units digit is 0, then the other three digits must be a 5 and two digits selected from 1, 2, 3, and 4. After the pair is selected, there are 6 possible ways to arrange them to form the number. So there are $6 \cdot 6 = 36$ possible numbers with units digit 0.
If the units digit is 5, then there are four choices (1, 2, 3, 4) for the thousands digit and there are $4 \cdot 3 = 12$ ways to complete the number. So there are $4 \cdot 12 = 48$ possible numbers with units digit 5.
Together there are $36 + 48 = 84$ possible numbers.

Difficulty: Hard
SMP-CCSS: 2, 7
Classification: Fundamental principle of counting

In how many ways can $10{,}001$ be written as the sum of two primes?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

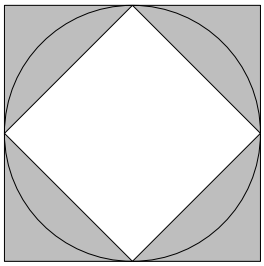
2011 AMC 8, Problem #24—
"If the sum of two numbers is odd, what are the two numbers? "

Solution

Answer (A): If the sum of two numbers is odd, one number must be even and the other number must be odd. Because all primes except 2 are odd, 2 must be one of the summands. Because $10{,}001 = 2 + 9999$, and $9999 = 9 \cdot 1111$ is not prime, there are no solutions.

Difficulty: Medium Hard
SMP-CCSS: 2, 3, 7
CCSS-M: None

A circle with radius 1 is inscribed in a square and circumscribed about another square as shown. Which fraction is closest to the ratio of the circle's shaded area to the shaded area between the two squares?

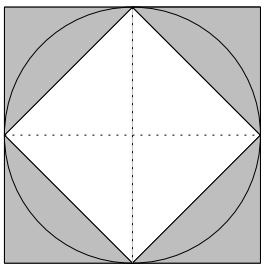


(A) $\frac{1}{2}$ **(B)** 1 **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** $\frac{5}{2}$

2011 AMC 8, Problem #25— "What is the area of a circle of radius 1? "

Solution

Answer (A): The area of a circle of radius 1 is $\pi(1)^2 = \pi$. The side length of the big square is the diameter of the circle, which is 2, so its area is $2^2 = 4$. The big square can be divided into 8 congruent triangles, and the shaded area is made up of 4 of those triangles. The shaded area is half the area of the big square, which is 2. The requested ratio of the two shaded areas is $\frac{\pi-2}{2} \approx \frac{3.14-2}{2} \approx \frac{1}{2}$.



Difficulty: Medium Hard
SMP-CCSS: 2, 7
CCSS-M: 7.G4, 7.G6