At Euclid Middle School the mathematics teachers are Miss Germain, Mr. Newton, and Mrs. Young. There are 11 students in Miss Germain's class, 8 students in Mr. Newton's class, and 9 students in Mrs. Young's class taking the AMC 8 Contest this year. How many mathematics students at Euclid Middle School are taking the contest?

(A) 26 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 30

2010 AMC 8, Problem #1— "How many students in each class?"

Solution

Answer (C): The total number of students taking the test is $11 + 8 + 9 = 28$.

Difficulty: Easy

CCSS-M: 6.NS.3 Fluently add, subtract, multiply an divide multi-digit decimals using the standard algorithm for each operation.

If $a * b = \frac{a \times b}{a+b}$ for a, b positive integers, then what is $5 * 10$?

(A) $\dfrac{3}{10}$ **(B)** 1 **(C)** 2 **(D)** $\dfrac{10}{3}$ **(E)** 50

2010 AMC 8, Problem #2— "Pay attention to the order of operations."

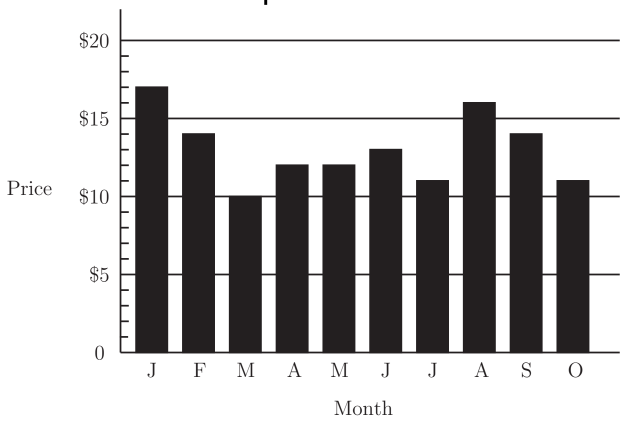
Solution

Answer (D): $5 * 10 = \frac{5 \times 10}{5+10} = \frac{50}{15} = \frac{5 \times 10}{5 \times 3} = \frac{10}{3}$.

Difficulty: Medium Easy

CCSS-M: 6.EE.2 Write, read and evaluate expressions in which letters stand for numbers.

The graph shows the price of five gallons of gasoline during the first ten months of the year. By what percent is the highest price more than the lowest price?



(A) 50 **(B)** 62 **(C)** 70 **(D)** 89 **(E)** 100

2010 AMC 8, Problem #3— "Read the heights on the bar chart carefully."

Solution

Answer (C): The highest price in January was $17 and the lowest in March was $10. The $17 price was $7 more than the $10 price, and 7 is 70% of 10.

Difficulty: Medium Easy

CCSS-M: 6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

What is the sum of the mean, median, and mode of the numbers $2, 3, 0, 3, 1, 4, 0, 3$?

(A) 6.5 **(B)** 7 **(C)** 7.5 **(D)** 8.5 **(E)** 9

2010 AMC 8, Problem #4—
"What are the definitions of mean, median, mode?"

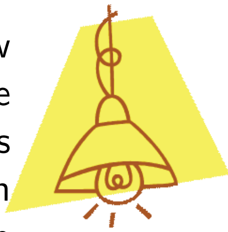
Solution

Answer (C): Arrange the numbers in increasing order: $0, 0, 1, 2, 3, 3, 3, 4$. The mean is the sum divided by 8, or $\frac{16}{8} = 2$. The median is halfway between 2 and 3, or 2.5. The mode is 3, because there are more 3's than any other number. The sum of the mean, median, and mode is $2 + 2.5 + 3 = 7.5$.

Difficulty: Medium

CCSS-M: 6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Alice needs to replace a light bulb located 10 centimeters below the ceiling in her kitchen. The ceiling is 2.4 meters above the floor. Alice is 1.5 meters tall and can reach 46 centimeters above the top of her head. Standing on a stool, she can just reach the light bulb. What is the height of the stool, in centimeters?



(A) 32 **(B)** 34 **(C)** 36 **(D)** 38 **(E)** 40

2010 AMC 8, Problem #5—
"How far above Alice's head is the ceiling?"

Solution

Answer (B): The ceiling is $2.4 - 1.5 = 0.9$ meters $= 90$ centimeters above Alice's head. She can reach 46 centimeters above the top of her head, and the light bulb is 10 centimeters below the ceiling, so the stool is $90 - 46 - 10 = 34$ centimeters high.

Difficulty: Medium Easy

CCSS-M: 7.NS.1 Represent addition and subtraction on a horizontal or vertical line diagram.

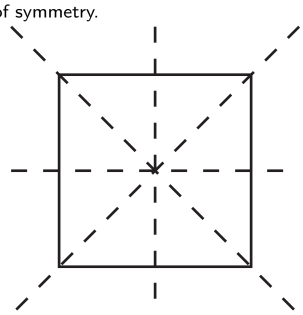
Which of the following figures has the greatest number of lines of symmetry?

(A) equilateral triangle **(B)** non-square rhombus
(C) non-square rectangle **(D)** isosceles trapezoid
(E) square

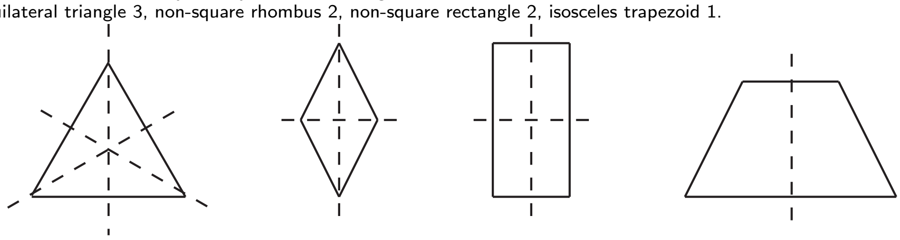
2010 AMC 8, Problem #6— "Draw each figure with its lines of symmetry."

Solution

Answer (E): A square has four lines of symmetry.



The number of lines of symmetry for the other figures are:
equilateral triangle 3, non-square rhombus 2, non-square rectangle 2, isosceles trapezoid 1.



Difficulty: Medium Easy

CCSS-M: 8.G.1. Verify experimentally the properties of rotations, reflections, and translations.

Using only pennies, nickels, dimes, and quarters, what is the smallest number of coins Freddie would need so he could pay any amount of money less than a dollar?

(A) 6 **(B)** 10 **(C)** 15 **(D)** 25 **(E)** 99

2010 AMC 8, Problem #7— "What is the fewest number of pennies needed?"

Solution

Answer (B): Four pennies are needed to make small change. Adding two nickels means that change up to fourteen cents can be made as efficiently as possible. Adding a dime extends the efficiency up to 24 cents. Adding three quarters permits any amount of change up to $\$0.99$.

$$4 \text{ pennies} + 2 \text{ nickels} + 1 \text{ dime} + 3 \text{ quarters} = 10 \text{ coins worth } \$0.99$$

Note that 4 pennies, 1 nickel, 2 dimes, and 3 quarters also satisfies the requirements. For 10 to be the smallest number of coins, one must prove that 9 coins will not work. Freddie will still need at least 4 pennies and 1 nickel to make any amount up to 9. Freddie will still need at least 1 more nickel and 1 dime to make any amount up to 24. If Freddie chose only 2 quarters, then he would need at least 9 other coins (4 dimes, 1 nickel, and 4 pennies) to reach 99 cents. He would need at least 11 coins. Thus 10 is the smallest number of coins as shown above.

Difficulty: Medium Hard

CCSS-M: 6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

As Emily is riding her bicycle on a long straight road, she spots Emerson skating in the same direction $1/2$ mile in front of her. After she passes him, she can see him in her rear view mirror until he is $1/2$ mile behind her. Emily rides at a constant rate of 12 miles per hour, and Emerson skates at a constant rate of 8 miles per hour. For how many minutes can Emily see Emerson?

(A) 6 **(B)** 8 **(C)** 12 **(D)** 15 **(E)** 16

2010 AMC 8, Problem #8— "What is the rate of gain of Emily on Emerson?"

Solution

Answer (D): Emily gains on Emerson at the rate of $12 - 8 = 4$ miles per hour. To get from $\frac{1}{2}$ mile behind Emerson to $\frac{1}{2}$ mile in front of him, she must gain 1 mile on him. This takes $\frac{1}{4}$ hour, which is 15 minutes.

OR

Make a chart that shows the position of Emily and Emerson every 5 minutes. Note that for every 5 minutes Emily rides $\frac{12}{60}$ miles per minute $\times$ 5 minutes $= 1$ mile, and Emerson skates $\frac{8}{12}$ mile. Eventually, Emily will be $\frac{1}{2}$ mile ahead of Emerson. Notice that happens after 15 minutes.

Time (minutes)	0	5	10	15
Emily's Distance (miles)	0	1	2	3
Emerson's Distance (miles)	$\frac{1}{2}$	$1\frac{1}{6}$	$1\frac{5}{6}$	$2\frac{1}{2}$

Difficulty: Medium Hard

CCSS-M: 6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. Solve unit rate problems including those involving unit pricing and constant speed.

Ryan got 80% of the problems correct on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all the problems did Ryan answer correctly?

(A) 63 **(B)** 75 **(C)** 80 **(D)** 84 **(E)** 86

2010 AMC 8, Problem #9— "What is Ryan's total points for all three tests?"

Solution

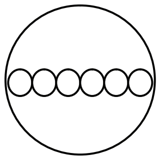
Answer (D): The three tests contain a total of 75 problems. Ryan received 80% of $25 = 20$, 90% of $40 = 36$, and 70% of $10 = 7$. Ryan correctly answered $20 + 36 + 7$ problems, for a total of 63 problems. The percent of problems Ryan answered correctly was:

$$\frac{63}{75} = \frac{21}{25} = \frac{21 \cdot 4}{25 \cdot 4} = \frac{84}{100} = 84\%$$

Difficulty: Medium

CCSS-M: 7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Six pepperoni circles will exactly fit across the diameter of a 12-inch pizza when placed as shown. If a total of 24 circles of pepperoni are placed on this pizza without overlap, what fraction of the pizza is covered by pepperoni?



(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{7}{8}$

2010 AMC 8, Problem #10—
"What's the diameter of each pepperoni slice?"

Solution

Answer (B): If six pepperonis fit across the diameter, then each pepperoni circle has a diameter of 2 inches and a radius of 1 inch. The area of each pepperoni is $\pi(1)^2 = \pi$ square inches. The 24 pepperoni circles cover 24π square inches of the pizza. The area of the pizza is $\pi(6)^2 = 36\pi$ square inches. The fraction of the pizza covered by pepperoni is $\frac{24\pi}{36\pi} = \frac{2}{3}$.

Difficulty: Medium Hard

CCSS-M: 7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems.

The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio $3 : 4$. In feet, how tall is the taller tree?

(A) 48 **(B)** 64 **(C)** 80 **(D)** 96 **(E)** 112

2010 AMC 8, Problem #11—
"Divide the sum of the heights of the trees into equal parts."

Solution

Answer (B): The sum of the heights of the two trees can be divided into 7 parts where one part is 16 feet. The taller tree has 4 parts so its height is $4 \times 16 = 64$ feet.

Difficulty: Medium Easy

CCSS-M: 6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed from the bag so that 75% of the remaining balls are red?

(A) 25 **(B)** 50 **(C)** 75 **(D)** 100 **(E)** 150

2010 AMC 8, Problem #12— "How many red and blue balls are there?"

Solution

Answer (D): The number of blue balls in the bag is 20% of 500, which is $(0.20)(500) = 100$. After some red balls are removed, the 100 blue balls must be 25%, or $\frac{1}{4}$, of the number in the bag. There must be $(4)(100) = 400$ balls in the bag, so $500 - 400 = 100$ red balls must be removed.

OR

Let x represent the number of red balls removed. Remember that $0.80(500) = 400$ is the number of red balls that were originally in the bag. Setup and solve the proportion:

$$75\% = \frac{3}{4} = \frac{400 - x}{500 - x}$$
$$3(500 - x) = 4(400 - x)$$
$$1500 - 3x = 1600 - 4x$$

The number of red balls removed is $x = 100$.

Difficulty: Medium Hard

CCSS-M: 6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

The lengths of the sides of a triangle measured in inches are three consecutive integers. The length of the shortest side is 30% of the perimeter. What is the length of the longest side?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

2010 AMC 8, Problem #13— "Make a table of triples of consecutive integers, try the possibilities."

Solution

Answer (E): One strategy is to try the choices:

$5+6+7=18;$	$5 \neq 30\%$ of 18
$6+7+8=21;$	$6 \neq 30\%$ of 21
$7+8+9=24;$	$7 \neq 30\%$ of 24
$8+9+10=27;$	$8 \neq 30\%$ of 27
$9+10+11=30;$	$9 = 30\%$ of 30

If the shortest side is 9, then the longest side is 11.

OR

Let the three consecutive integers be side lengths $x, x-1$, and $x-2$.

$$\begin{aligned} x-2 &= 0.3(x+x-1+x-2) \\ x-2 &= 0.3(3x-3) \\ x-2 &= 0.9x-0.9 \\ 0.1x &= 1.1 \\ x &= 11 \end{aligned}$$

The longest side is 11.

Difficulty: Medium Hard

CCSS-M: 6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true?

What is the sum of the prime factors of 2010 ?

(A) 67 **(B)** 75 **(C)** 77 **(D)** 201 **(E)** 210

2010 AMC 8, Problem #14— "Find the prime factors of 2010."

Solution

Answer (C): The prime factors of 2010 are: 2, 3, 5, 67
The sum of the prime factors is $2 + 3 + 5 + 67 = 77$.

Difficulty: Medium Hard

CCSS-M: 4.OA.4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

A jar contains five different colors of gum drops: 30% are blue, 20% are brown, 15% are red, 10% are yellow, and the other 30 gum drops are green. If half of the blue gum drops are replaced by brown gum drops, how many of the gum drops will be brown?

(A) 35 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 64

2010 AMC 8, Problem #15— "What percentage of the gum drops are green?"

Solution

Answer (C): The 30 green gum drops are $100\% - (30 + 20 + 15 + 10)\% = 25\%$ of the total gum drops, so there are 120 gum drops in the jar. The number of blue gum drops is 30% of 120, which is 36, and the number of brown gum drops is 20% of 120, which is 24. After half the blue gum drops are replaced by brown ones, the number of brown gum drops is $24 + \frac{1}{2}(36) = 42$.

Difficulty: Medium Hard

CCSS-M: 6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems. Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle?

(A) $\frac{\sqrt{\pi}}{2}$ **(B)** $\sqrt{\pi}$ **(C)** π **(D)** 2π **(E)** π^2

2010 AMC 8, Problem #16— "What's the area of the circle?"

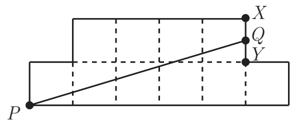
Solution

Answer (B): Let the radius of the circle be 1. Then the area of the circle is $\pi(1)^2 = \pi$. The area of the square is π, so its side length $\sqrt{\pi}$. The ratio of the side length of the square to the radius of the circle is $\frac{\sqrt{\pi}}{1} = \sqrt{\pi}$.
Note: The "squaring of a circle" is a classical problem. In the latter part of the 19th century it was proven that a square having an area equal to that of a given circle cannot be constructed with the standard tools of straightedge and compasss because it is impossible to construct a transcendental number, e. g. $\sqrt{\pi}$.

Difficulty: Hard

CCSS-M: 7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems.

The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\frac{XQ}{QY}$?



(A) $\dfrac{2}{5}$ **(B)** $\dfrac{1}{2}$ **(C)** $\dfrac{3}{5}$ **(D)** $\dfrac{2}{3}$ **(E)** $\dfrac{3}{4}$

2010 AMC 8, Problem #17— "What is the actual area below $\overline{PQ}$?"

Solution

Answer (D): The area below $\overline{PQ}$ is

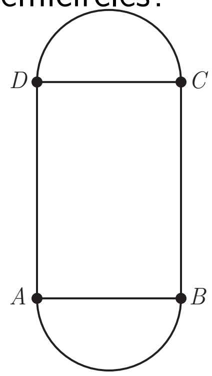
$$\begin{aligned} 1 + \frac{1}{2} \cdot 5 \cdot (1 + QY) &= 5 \\ \frac{5}{2} \cdot (1 + QY) &= 4 \\ 1 + QY &= \frac{8}{5} \\ QY &= \frac{3}{5} \end{aligned}$$

Then $XQ = 1 - QY = 1 - \frac{3}{5} = \frac{2}{5}$, so $\frac{XQ}{QY} = \frac{2/5}{3/5} = \frac{2}{3}$.

Difficulty: Hard

CCSS-M: 7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

A decorative window is made up of a rectangle with semicircles on either end. The ratio of AD to AB is $3 : 2$ and $AB = 30$ inches. What is the ratio of the area of the rectangle to the combined areas of the semicircles?



(A) $2 : 3$ **(B)** $3 : 2$ **(C)** $6 : \pi$ **(D)** $9 : \pi$ **(E)** $30 : \pi$

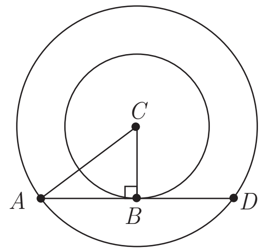
2010 AMC 8, Problem #18— "What is the area of the rectangle?"

Solution

Answer (C): The given ratio implies that $AD = \frac{3}{2}AB = \frac{3}{2} \cdot 30 = 45$ inches, so the area of the rectangle is $AB \cdot AD = 30 \cdot 45$ square inches. The 2 semicircles make 1 circle with radius $= 15$ inches. The area of the circle is $15^2\pi$ square inches. The ratio of the areas is $\frac{30 \cdot 45}{15 \cdot 15\pi} = \frac{6}{\pi}$.

Difficulty: Medium Hard
CCSS-M: 7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems.

The two circles pictured have the same center C. Chord $\overline{AD}$ is tangent to the inner circle at B, AC is 10, and chord $\overline{AD}$ has length 16. What is the area between the two circles?

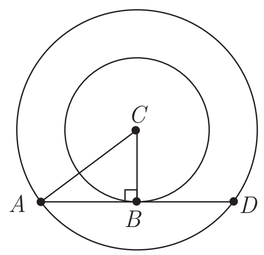


(A) 36π **(B)** 49π **(C)** 64π **(D)** 81π **(E)** 100π

2010 AMC 8, Problem #19— "Make use of the Pythagorean Theorem."

Solution

Answer (C): In the right triangle ABC, AB is 8. By the Pythagorean Theorem, $8^2 + BC^2 = 10^2$ so $BC = 6$. The area of the outer circle is $10^2\pi = 100\pi$ and the area of the inner circle is $6^2\pi = 36\pi$. The area between the circles is $100\pi - 36\pi = 64\pi$.



Difficulty: Medium Hard

CCSS-M: 8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

In a room, $\frac{2}{5}$ of all the people are wearing gloves, and $\frac{3}{4}$ of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and gloves?

(A) 3 **(B)** 5 **(C)** 8 **(D)** 15 **(E)** 20

2010 AMC 8, Problem #20— "The number of people is the multiple of what numbers?"

Solution

Answer (A): Because $\frac{2}{5}$ and $\frac{3}{4}$ of the people in the room are whole numbers, the number of people in the room is a multiple of both 5 and 4. The least common multiple of 4 and 5 is 20, so the minimum number of people in the room is 20. If $\frac{2}{5}$ of 20 people are wearing gloves, then 8 people are wearing gloves. If $\frac{3}{4}$ of 20 people are wearing hats, then 15 are wearing hats. The minimum number wearing gloves and hats occurs if the 5 not wearing hats are each wearing gloves. This leaves $8 - 5 = 3$ people wearing both gloves and hats.

OR

If 8 are wearing gloves and 15 are wearing hats, then $8 + 15$ are wearing gloves and/or hats. There is a minimum of 20 people in the room, so $23 - 20 = 3$ people are wearing both a hat and gloves.

Difficulty: Medium Hard

CCSS-M: 6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.

Hui is an avid reader. She bought a copy of the bestseller *Math is Beautiful*. On the first day, Hui read $\frac{1}{5}$ of the pages plus 12 more, and on the second day she read $\frac{1}{4}$ of the remaining pages, plus 15 pages. On the third day, she read $\frac{1}{3}$ of the remaining pages, plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

(A) 120 **(B)** 180 **(C)** 240 **(D)** 300 **(E)** 360

2010 AMC 8, Problem #21—

"Set up a chart of pages read, working backwards through the days."

Solution

Answer (C): Reason backward as follows. On the third day, Hui reads $\frac{1}{3}$ of the remaining pages plus 18 more, leaving her with 62 pages left to read. This means that $62 + 18 = 80$ is $\frac{2}{3}$ of the number of pages remaining at the end of the second day. She had $\frac{3}{2} \times 80 = 120$ pages left to read at the end of the second day. On the second day she read $\frac{1}{4}$ of the remaining pages plus 15 more, so $120 + 15 = 135$ is $\frac{3}{4}$ of the number of pages remaining at the end of the first day. She had $\frac{4}{3} \times 135 = 180$ pages left to read at the end of the first day. On the first day she read $\frac{1}{5}$ of the pages plus 12 more. So $180 + 12 = 192$ is $\frac{4}{5}$ of the number of pages in the book. The total number of pages is $\frac{5}{4} \times 192 = 240$.

OR

Setup a chart working backwards through the days, to show the number of pages that Hui has left to read each day.

Day	Extra Pages Read	Fraction Read	Fraction Left	Calculation	Pages to be Read
4	0				62
3	18	$1/3$	$2/3$	$(62+18)/(2/3)$	120
2	15	$1/4$	$3/4$	$(120+15)/(3/4)$	180
1	12	$1/5$	$4/5$	$(180+12)/(4/5)$	240

Difficulty: Medium Hard

CCSS-M: 7.EE.3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.

The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2010 AMC 8, Problem #22— "What is the original number to start with?"

Solution

Answer (E): Take 452 as the original number, and subtract 254. The difference is 198 and the units digit is 8. The same result will be obtained with any number that meets the criteria.

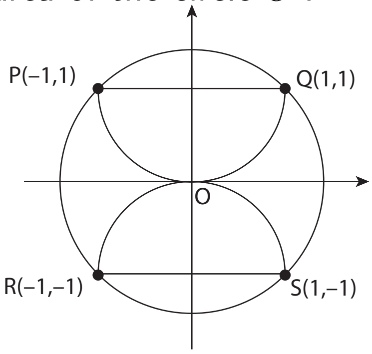
OR

The original number is $100a + 10b + c$, with $a - c = 2$. The reversed number will be $100c + 10b + a$. The difference is $99a - 99c = 99(a - c) = 99(2) = 198$. So the units digit is 8.

Difficulty: Medium Hard

CCSS-M: 5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Semicircles POQ and ROS pass through the center of circle O. What is the ratio of the combined areas of the two semicircles to the area of the circle O?



(A) $\frac{\sqrt{2}}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{\pi}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{\sqrt{2}}{2}$

2010 AMC 8, Problem #23— "Take advantage of the Pythagorean Theorem."

Solution

Answer (B): By the Pythagorean Theorem, the radius OQ of circle O is $\sqrt{2}$. Given the coordinates P, Q, R, and S, the diameters $\overline{PQ}$ and $\overline{RS}$ of the semicircles have length 2. So the areas of the two semicircles will equal the area of a circle of radius 1. Thus the desired ratio is $\frac{\pi \cdot 1^2}{\pi(\sqrt{2})^2} = \frac{1}{2}$.

Difficulty: Medium Hard

CCSS-M: 8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

What is the correct ordering of the three numbers 10^8, 5^{12}, and 2^{24}?

(A) $2^{24} < 10^8 < 5^{12}$ **(B)** $2^{24} < 5^{12} < 10^8$
(C) $5^{12} < 2^{24} < 10^8$ **(D)** $10^8 < 5^{12} < 2^{24}$
(E) $10^8 < 2^{24} < 5^{12}$

2010 AMC 8, Problem #24—
"Express each number either in factors or with other bases."

Solution

Answer (A): $2^{24} = (2^8) \cdot (2^{16}) = (2^8) \cdot (4^8) < (2^8) \cdot (5^8) = 10^8 = (4^4) \cdot (5^8) < (5^4) \cdot (5^8) = 5^{12}$.

Difficulty: Hard
CCSS-M: 6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.

Every day at school, Jo climbs a flight of 6 stairs. Jo can take stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2 stairs. In how many ways can Jo climb the stairs?

(A) 13 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

2010 AMC 8, Problem #25— "Systematically list all the possibilities."

Solution

Answer (E): Jo can climb 1 stair in 1 way, 2 stairs in 2 ways: $1+1$ or 2, and 3 stairs in 4 ways: $1+1+1$, $1+2$, $2+1$ or 3. Jo can start a flight of 4 stairs with 1 stair, leaving 3 stairs to go and 4 ways to climb them, or with 2 stairs, leaving 2 stairs to go and 2 ways to climb them or with 3 stairs, leaving 1 to go and 1 way to climb it. This means there are $1+2+4=7$ ways to climb 4 stairs. By the same argument, there must be $2+4+7=13$ ways to climb 5 stairs and $4+7+13=24$ ways to climb 6 stairs.

OR

Making a systematic list is another approach.

Number of stair moves	Stair move sequence	Count
6	1-1-1-1-1-1	1
5	1-1-1-1-2; 1-1-1-2-1; 1-1-2-1-1; 1-2-1-1-1; and 2-1-1-1-1	5
4	1-1-1-3; 1-1-3-1; 1-3-1-1; 3-1-1-1; 1-1-2-2; 1-2-1-2; 1-2-2-1; 2-1-1-2; 2-1-2-1; and 2-2-1-1	10
3	1-2-3; 1-3-2; 2-1-3; 2-3-1; 3-1-2; 3-2-1; and 2-2-2	7
2	3-3	1

The number of ways Jo can climb the stairs is $1+5+10+7+1=24$.

Difficulty: Hard

CCSS-M: 7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. Represent sample spaces for compound events using methods such as organized lists.