

# MAA

MATHEMATICAL ASSOCIATION OF AMERICA

## American Mathematics Competitions

26<sup>th</sup> Annual

# AMC 8

American Mathematics Contest 8

Tuesday, November 16, 2010

## INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record your information on the answer form.
8. You will have 40 minutes to complete the test once your proctor tells you to begin.
9. When you finish the exam, *sign your name* in the space provided on the answer form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 8 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of individual problems in paper or electronic form including posting on web pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

2010

# AMC 8



**DO NOT OPEN UNTIL TUESDAY, NOVEMBER 16, 2010**

**\*\*ADMINISTRATION ON AN EARLIER DATE  
WILL DISQUALIFY YOUR SCHOOL'S RESULTS\*\***

1. PLEASE READ THE TEACHERS' MANUAL BEFORE NOVEMBER 16, 2010. All rules and instructions needed to administer this exam are contained in the manual. You will not need anything from inside this package until November 16.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 8 CERTIFICATION FORM that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be sent by trackable mail to the AMC office no later than 24 hours following the exam.
4. THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.
5. The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, email, internet or media of any type during this period is a violation of the competition rules.

## The American Mathematics Competitions

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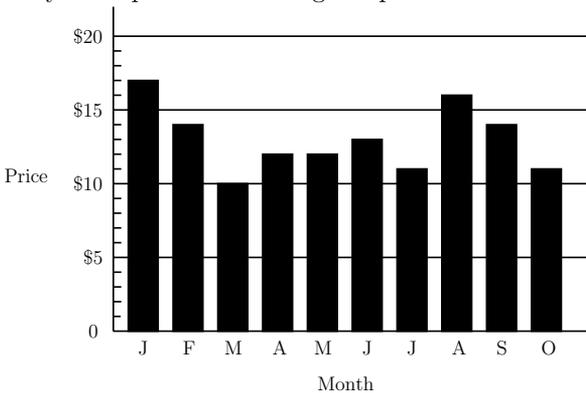
1. At Euclid Middle School the mathematics teachers are Miss Germain, Mr. Newton, and Mrs. Young. There are 11 students in Miss Germain's class, 8 students in Mr. Newton's class, and 9 students in Mrs. Young's class taking the AMC 8 Contest this year. How many mathematics students at Euclid Middle School are taking the contest?

(A) 26    (B) 27    (C) 28    (D) 29    (E) 30

2. If  $a * b = \frac{a \times b}{a + b}$  for  $a, b$  positive integers, then what is  $5 * 10$ ?

(A)  $\frac{3}{10}$     (B) 1    (C) 2    (D)  $\frac{10}{3}$     (E) 50

3. The graph shows the price of five gallons of gasoline during the first ten months of the year. By what percent is the highest price more than the lowest price?



(A) 50    (B) 62    (C) 70    (D) 89    (E) 100

4. What is the sum of the mean, median, and mode of the numbers 2, 3, 0, 3, 1, 4, 0, 3?

(A) 6.5    (B) 7    (C) 7.5    (D) 8.5    (E) 9

5. Alice needs to replace a light bulb located 10 centimeters below the ceiling in her kitchen. The ceiling is 2.4 meters above the floor. Alice is 1.5 meters tall and can reach 46 centimeters above the top of her head. Standing on a stool, she can just reach the light bulb. What is the height of the stool, in centimeters?

(A) 32    (B) 34    (C) 36    (D) 38    (E) 40



6. Which of the following figures has the greatest number of lines of symmetry?

- (A) equilateral triangle      (B) non-square rhombus  
 (C) non-square rectangle      (D) isosceles trapezoid      (E) square

7. Using only pennies, nickels, dimes, and quarters, what is the smallest number of coins Freddie would need so he could pay any amount of money less than a dollar?

- (A) 6      (B) 10      (C) 15      (D) 25      (E) 99



8. As Emily is riding her bicycle on a long straight road, she spots Emerson skating in the same direction  $\frac{1}{2}$  mile in front of her. After she passes him, she can see him in her rear view mirror until he is  $\frac{1}{2}$  mile behind her. Emily rides at a constant rate of 12 miles per hour, and Emerson skates at a constant rate of 8 miles per hour. For how many minutes can Emily see Emerson?

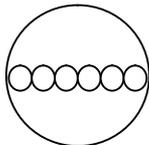


- (A) 6      (B) 8      (C) 12      (D) 15      (E) 16

9. Ryan got 80% of the problems correct on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all the problems did Ryan answer correctly?

- (A) 63      (B) 75      (C) 80      (D) 84      (E) 86

10. Six pepperoni circles will exactly fit across the diameter of a 12-inch pizza when placed as shown. If a total of 24 circles of pepperoni are placed on this pizza without overlap, what fraction of the pizza is covered by pepperoni?



- (A)  $\frac{1}{2}$       (B)  $\frac{2}{3}$       (C)  $\frac{3}{4}$       (D)  $\frac{5}{6}$       (E)  $\frac{7}{8}$

11. The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio 3 : 4. In feet, how tall is the taller tree?

- (A) 48      (B) 64      (C) 80      (D) 96      (E) 112

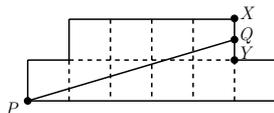
12. Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed from the bag so that 75% of the remaining balls are red?
- (A) 25    (B) 50    (C) 75    (D) 100    (E) 150
13. The lengths of the sides of a triangle measured in inches are three consecutive integers. The length of the shortest side is 30% of the perimeter. What is the length of the longest side?
- (A) 7    (B) 8    (C) 9    (D) 10    (E) 11
14. What is the sum of the prime factors of 2010?
- (A) 67    (B) 75    (C) 77    (D) 201    (E) 210

15. A jar contains five different colors of gum drops: 30% are blue, 20% are brown, 15% are red, 10% are yellow, and the other 30 gum drops are green. If half of the blue gum drops are replaced by brown gum drops, how many of the gum drops will be brown?



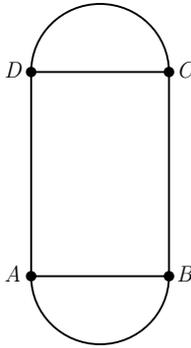
- (A) 35    (B) 36    (C) 42    (D) 48    (E) 64
16. A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle?
- (A)  $\frac{\sqrt{\pi}}{2}$     (B)  $\sqrt{\pi}$     (C)  $\pi$     (D)  $2\pi$     (E)  $\pi^2$

17. The diagram shows an octagon consisting of 10 unit squares. The portion below  $\overline{PQ}$  is a unit square and a triangle with base 5. If  $\overline{PQ}$  bisects the area of the octagon, what is the ratio  $\frac{XQ}{QY}$ ?

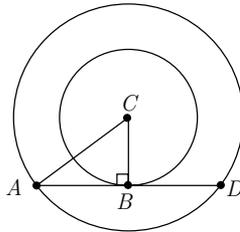


- (A)  $\frac{2}{5}$     (B)  $\frac{1}{2}$     (C)  $\frac{3}{5}$     (D)  $\frac{2}{3}$     (E)  $\frac{3}{4}$

18. A decorative window is made up of a rectangle with semicircles on either end. The ratio of  $AD$  to  $AB$  is  $3 : 2$  and  $AB = 30$  inches. What is the ratio of the area of the rectangle to the combined areas of the semicircles?



- (A)  $2 : 3$     (B)  $3 : 2$     (C)  $6 : \pi$     (D)  $9 : \pi$     (E)  $30 : \pi$
19. The two circles pictured have the same center  $C$ . Chord  $\overline{AD}$  is tangent to the inner circle at  $B$ ,  $AC$  is 10, and chord  $\overline{AD}$  has length 16. What is the area between the two circles?



- (A)  $36\pi$     (B)  $49\pi$     (C)  $64\pi$     (D)  $81\pi$     (E)  $100\pi$
20. In a room,  $\frac{2}{5}$  of all the people are wearing gloves, and  $\frac{3}{4}$  of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and gloves?

- (A) 3    (B) 5    (C) 8    (D) 15    (E) 20





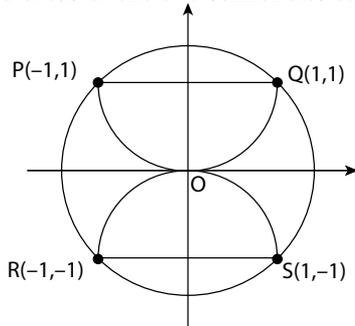
21. Hui is an avid reader. She bought a copy of the best-seller *Math is Beautiful*. On the first day, Hui read  $\frac{1}{5}$  of the pages plus 12 more, and on the second day she read  $\frac{1}{4}$  of the remaining pages, plus 15 pages. On the third day, she read  $\frac{1}{3}$  of the remaining pages, plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

(A) 120 (B) 180 (C) 240 (D) 300 (E) 360

22. The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

23. Semicircles  $POQ$  and  $ROS$  pass through the center of circle  $O$ . What is the ratio of the combined areas of the two semicircles to the area of the circle  $O$ ?



(A)  $\frac{\sqrt{2}}{4}$  (B)  $\frac{1}{2}$  (C)  $\frac{2}{\pi}$  (D)  $\frac{2}{3}$  (E)  $\frac{\sqrt{2}}{2}$

24. What is the correct ordering of the three numbers  $10^8$ ,  $5^{12}$ , and  $2^{24}$ ?

(A)  $2^{24} < 10^8 < 5^{12}$  (B)  $2^{24} < 5^{12} < 10^8$  (C)  $5^{12} < 2^{24} < 10^8$   
 (D)  $10^8 < 5^{12} < 2^{24}$  (E)  $10^8 < 2^{24} < 5^{12}$

25. Every day at school, Jo climbs a flight of 6 stairs. Jo can take stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2 stairs. In how many ways can Jo climb the stairs?

- (A) 13      (B) 18      (C) 20      (D) 22      (E) 24



## SOLUTIONS

Your School Manager has been sent at least one copy of the 2010 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

## WRITE TO US

*Comments about the problems and solutions for this AMC 8 should be addressed to:*

**Ms. Margie Raub Hunt**, AMC 8 Chair  
2169 Madero Dr., The Villages, FL 32159

*Comments about administrative arrangements should be addressed to:*

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## AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute, multiple choice contests. All schools participating in the AMC 8 receive a brochure and registration form for the 2011 AMC 10. Schools with high scoring students on the AMC 8 should consider administering the AMC 10. The best way to prepare for these contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

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## PUBLICATIONS

A complete listing of the current publications for sale can be found on our web site:  
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