Bridget bought a bag of apples at the grocery store. She gave half of the apples to Ann. Then she gave Cassie 3 apples, keeping 4 apples for herself. How many apples did Bridget buy?

(A) 3 **(B)** 4 **(C)** 7 **(D)** 11 **(E)** 14

2009 AMC 8, Problem #1—
"The problem can be solved if work backwards."

Solution

Answer (E): Work backwards. Bridget had 7 apples before she gave Cassie 3 apples. These 7 apples were half of Bridget's 14 original apples.

OR

Let $B =$ Bridget's original number of apples.

$$\frac{B}{2} - 3 = 4$$
$$\frac{B}{2} = 7$$
$$B = 14$$

So Bridget originally had 14 apples.

Difficulty: Easy

NCTM Standard: Algebra Standards for Grades 6–8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.

Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic > Arithmetic

On average, for every 4 sports cars sold at the local car dealership, 7 sedans are sold. The dealership predicts that it will sell 28 sports cars next month. How many sedans does it expect to sell?

(A) 7 **(B)** 32 **(C)** 35 **(D)** 49 **(E)** 112

2009 AMC 8, Problem #2—

"$28 = 7 \times 4$."

Solution

Answer (D): Let $s =$ number of sedans. Set up a proportion: $\frac{4}{7} = \frac{28}{s} = \frac{4(7)}{7(7)} = \frac{28}{49}$. So the dealership expects to sell 49 sedans.

OR

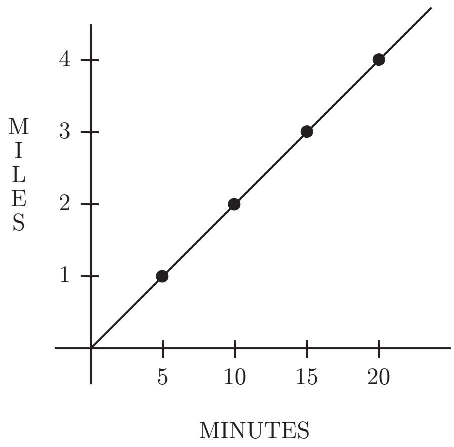
Because selling 4 sports cars corresponds to selling 7 sedans, 28 sports cars = 7(4 sports cars) corresponds to 7(7 sedans) = 49 sedans.

Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand and use ratios and proportions to represent quantitative relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Ratio

The graph shows the constant rate at which Suzanna rides her bike. If she rides a total of half an hour at the same speed, how many miles will she have ridden?



(A) 5 **(B)** 5.5 **(C)** 6 **(D)** 6.5 **(E)** 7

2009 AMC 8, Problem #3— "Find Suzanna's speed in the unit of "five minutes per mile"."

Solution

Answer (C): Suzanna rides at a constant rate of five minutes per mile. In 30 minutes there are six 5-minute intervals, so she travels six miles.

OR

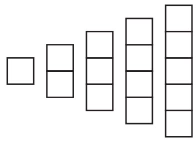
Suzanna rides 3 miles in 15 minutes, so she will ride 6 miles in 30 minutes.

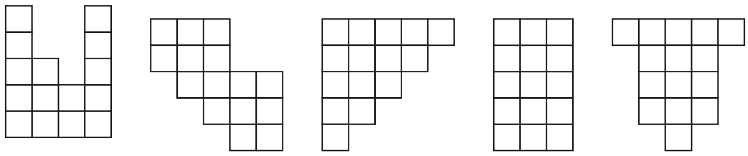
Difficulty: Easy

NCTM Standard: Algebra Standard for Grades 6–8: use graphs to analyze the nature of changes in quantities in linear relationships.

Mathworld.com Classification: Geometry > Line Geometry > Lines > Slope

The five pieces shown at right can be arranged to form four of the five figures below. Which figure **cannot** be formed?





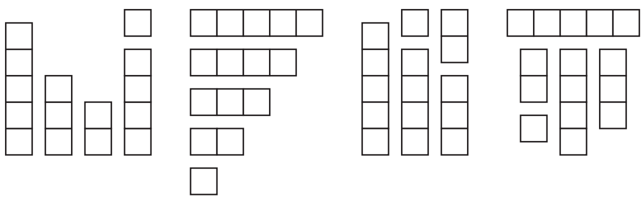
(A) (B) (C) (D) (E)

2009 AMC 8, Problem #4—

"Look carefully in each choice, check if all pieces are possible in each figure."

Solution

Answer (B): Figure B does not contain any 5-square-long piece. One solution is given for each of the other four figures. There are other solutions.



(A) (C) (D) (E)

Difficulty: Easy

NCTM Standard: Geometry Standard for Grades 6–8: describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.

Mathworld.com Classification: NEED CATEGORY!!!!!

A sequence of numbers starts with 1, 2 and 3. The fourth number of the sequence is the sum of the previous three numbers in the sequence: $1 + 2 + 3 = 6$. In the same way, every number after the fourth is the sum of the previous three numbers. What is the eighth number in the sequence?

(A) 11 **(B)** 20 **(C)** 37 **(D)** 68 **(E)** 99

2009 AMC 8, Problem #5— "Make a table to find all first 8 numbers in the sequence."

Solution

Answer (D): Make the list:

Position	1	2	3	4	5	6	7	8
Number	1	2	3	$1+2+3$ $=6$	$2+3+6$ $=11$	$3+6+11$ $=20$	$6+11+20$ $=37$	$11+20+37$ $=$ **68**

So the eighth number in the sequence is 68.

Difficulty: Easy

NCTM Standard: Algebra Standard for Grades 6–8: represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.

Mathworld.com Classification: Discrete Mathematics > Recurrence Equations > Recursive Sequence

Steve's empty swimming pool will hold $24{,}000$ gallons of water when full. It will be filled by 4 hoses, each of which supplies 2.5 gallons of water per minute. How many hours will it take to fill Steve's pool?

(A) 40 **(B)** 42 **(C)** 44 **(D)** 46 **(E)** 48

2009 AMC 8, Problem #6— "Together the hoses supply 10 gallons per minute to the pool."

Solution

Answer (A): Together the hoses supply 10 gallons per minute to the pool. The pool holds $24{,}000$ gallons, so it will take a total of $\frac{24{,}000 \text{ gallons}}{10 \text{ gallons/minute}} = 2400$ minutes. Because 2400 minutes equals 40 hours, it takes 40 hours to fill Steve's pool.

OR

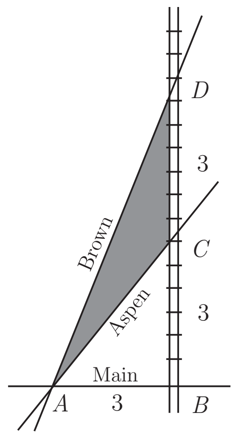
The hoses supply (10 gallons/minute)(60 minutes/hour) $=$ 600 gallons/hour. So it will take $\frac{24{,}000 \text{ gallons}}{600 \text{ gallons/hour}} = 40$ hours to fill Steve's pool.

Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 6–8: compute fluently and make reasonable estimates.

Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic > Arithmetic

The triangular plot of land ACD lies between Aspen Road, Brown Road and a railroad. Main Street runs east and west, and the railroad runs north and south. The numbers in the diagram indicate distances in miles. The width of the railroad track can be ignored. How many square miles are in the plot of land ACD?



(A) 2 **(B)** 3 **(C)** 4.5 **(D)** 6 **(E)** 9

2009 AMC 8, Problem #7—
"Area of $\triangle ACD$ = Area of $\triangle ABD$ - Area of $\triangle ABC$."

Solution

Answer (C): The area of $\triangle ABC$ is $\frac{1}{2}(3)(3) = \frac{9}{2}$ square miles. The area of $\triangle ABD = \frac{1}{2}(3)(6) = 9$ square miles. The shaded area is the area of $\triangle ABD$ minus the area of $\triangle ABC$, which is $9 - \frac{9}{2} = \frac{9}{2} = 4.5$ square miles.

OR

The base $\overline{CD}$ of $\triangle ACD$ is 3 miles. The altitude $\overline{AB}$ of $\triangle ACD$ is 3 miles. The area of $\triangle ACD$ is $\frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2} = 4.5$ squares miles.

Difficulty: Easy

NCTM Standard: Geometry Standard for Grades 6–8: use coordinate geometry to represent and examine the properties of geometric shapes.

Mathworld.com Classification: Geometry > Plane Geometry > Triangles > Triangle Properties > Triangle Area

The length of a rectangle is increased by 10% and the width is decreased by 10%. What percent of the old area is the new area?

(A) 90 **(B)** 99 **(C)** 100 **(D)** 101 **(E)** 110

2009 AMC 8, Problem #8— "Use variables to represent the length and width and observe the change in the product."

Solution

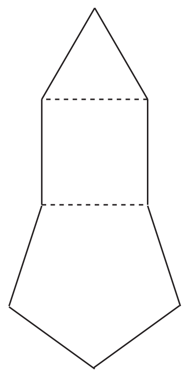
Answer (B): A rectangle with length L and width W has area LW. The new rectangle has area $(1.1)L \times (0.9)W = 0.99LW$. The new area $0.99LW$ is 99% of the old area.

Difficulty: Easy

NCTM Standard: Geometry Standard for Grades 6–8: describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.

Mathworld.com Classification: Geometry > Plane Geometry > Rectangles > Rectangle

Construct a square on one side of an equilateral triangle. On one non-adjacent side of the square, construct a regular pentagon, as shown. On a non-adjacent side of the pentagon, construct a regular hexagon. Continue to construct regular polygons in the same way, until you construct an octagon. How many sides does the resulting polygon have?



(A) 21 **(B)** 23 **(C)** 25 **(D)** 27 **(E)** 29

2009 AMC 8, Problem #9— "Make a table and add the appropriate number of sides."

Solution

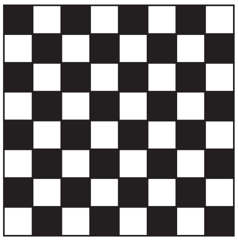
Answer (B): One side of the triangle and one side of the octagon will each touch one other polygon. Two sides of the other polygons will touch other polygons. Make a table and add the appropriate number of sides.

Number of sides	3	4	5	6	7	8
Number of sides that touch other polygons	1	2	2	2	2	1
Number of sides that don't	2	2	3	4	5	7

The resulting polygon has $2 + 2 + 3 + 4 + 5 + 7 = 23$ sides.

Difficulty: Easy
NCTM Standard: Geometry Standard for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
Mathworld.com Classification: Geometry > Plane Geometry > Polygons > Polygon

On a checkerboard composed of 64 unit squares, what is the probability that a randomly chosen unit square does **not** touch the outer edge of the board?



(A) $\frac{1}{16}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{49}{64}$

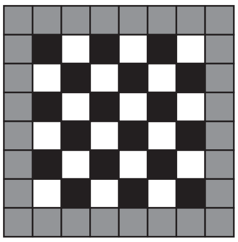
2009 AMC 8, Problem #10—

"There are $2 \cdot 8 + 2 \cdot 6 = 28$ unit squares on the outer edge out of the totaly 64 unit squares on the checkerboard."

Solution

Answer (D): The checkerboard has 64 unit squares. There are $2 \cdot 8 + 2 \cdot 6 = 28$ unit squares on the outer edge, and $64 - 28 = 36$ unit squares in the interior. Therefore the probability of choosing a unit square that does not touch the outer edge is $\frac{36}{64} = \frac{18}{32} = \frac{9}{16}$.

OR



There are $(8-2)^2 = 36$ unit squares in the interior. Therefore, the probability of choosing a unit square that is does not touch the outer edge is $\frac{36}{64} = \frac{18}{32} = \frac{9}{16}$.

Difficulty: Easy

NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: understand and apply basic concepts of probability.

Mathworld.com Classification: Probability and Statistics > Probability > Probability

The Amaco Middle School bookstore sells pencils costing a whole number of cents. Some seventh graders each bought a pencil, paying a total of $\$1.43$. Some of the 30 sixth graders each bought a pencil, and they paid a total of $\$1.95$. How many more sixth graders than seventh graders bought a pencil?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

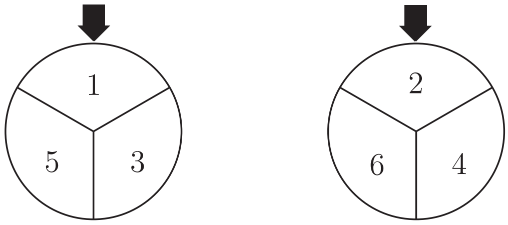
2009 AMC 8, Problem #11—
"Both 195 and 143 are multiples of the price of the pencil."

Solution

Answer (D): The number of sixth graders who bought a pencil is 195 divided by the cost of a pencil. Similarly the number of seventh graders who bought a pencil is 143 divided by the cost of a pencil. That means both 195 and 143 are multiples of the price of the pencil. Factor $195 = 1 \cdot 3 \cdot 5 \cdot 13$ and $143 = 1 \cdot 11 \cdot 13$. The only common divisors are 1 and 13. If a pencil cost 1 cent, then 195 sixth graders bought a pencil. However, there are only 30 sixth graders, so a pencil must cost 13 cents. Using that fact, $\frac{195}{13} - \frac{143}{13} = 15 - 11 = 4$ more sixth graders than seventh graders bought pencils.

Difficulty: Easy
NCTM Standard: Number and Operations Standard for Grades 6–8: use factors, multiples, prime factorization, and relatively prime numbers to solve problems.
Mathworld.com Classification: Number Theory > Divisors > Divisor

The two spinners shown are spun once and each lands on one of the numbered sectors. What is the probability that the sum of the numbers in the two sectors is prime?



(A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{7}{9}$ **(E)** $\frac{5}{6}$

2009 AMC 8, Problem #12— "Make a table for all possible outcomes."

Solution

Answer (D): Make a table.

	1	3	5
2	$1+2=3$	$3+2=5$	$5+2=7$
4	$1+4=5$	$3+4=7$	$5+4=9$
6	$1+6=7$	$3+6=9$	$5+6=11$

The table shows that seven of the nine equally likely events have prime numbers for their outcomes. So the probability of a prime outcome is $\frac{7}{9}$.

Difficulty: Easy

NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: understand and apply basic concepts of probability.

Mathworld.com Classification: Probability and Statistics > Probability > Probability

A three-digit integer contains one of each of the digits 1, 3 and 5. What is the probability that the integer is divisible by 5?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{5}{6}$

2009 AMC 8, Problem #13—
"The number is divisible by 5 only if it ends in 5."

Solution

Answer (B): There are 6 three-digit numbers possible using the digits 1, 3 and 5 once each: 135, 153, 315, 351, 513 and 531. Because the numbers divisible by 5 end in 0 or 5, only 135 and 315 are divisible by 5. The probability that the three-digit number is divisible by 5 is $\frac{2}{6} = \frac{1}{3}$.

OR

The number is equally likely to end in 1, 3 or 5. The number is divisible by 5 only if it ends in 5, so the probability is $\frac{1}{3}$.

Difficulty: Easy

NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: understand and apply basic concepts of probability.

Mathworld.com Classification: Probability and Statistics > Probability > Probability

Austin and Temple are 50 miles apart along Interstate 35. Bonnie drove from Austin to her daughter's house in Temple, averaging 60 miles per hour. Leaving the car with her daughter, Bonnie rode a bus back to Austin along the same route and averaged 40 miles per hour on the return trip. What was the average speed for the round trip, in miles per hour?

(A) 46 **(B)** 48 **(C)** 50 **(D)** 52 **(E)** 54

2009 AMC 8, Problem #14— "Average Speed = Total Distance ÷ Total time."

Solution

Answer (B): Find the time traveling to Temple by dividing the distance, 50 miles, by the rate, 60 miles per hour: $\frac{50}{60} = \frac{5}{6}$ hours. Find the time returning by dividing the distance, 50 miles, by the rate, 40 miles per hour: $\frac{50}{40} = \frac{5}{4}$ hours. Find the average speed for the round trip by dividing the total distance, $2 \cdot 50 = 100$ miles, by the total time, $\frac{5}{6} + \frac{5}{4} = \frac{10}{12} + \frac{15}{12} = \frac{25}{12}$ hours. The average speed is $\dfrac{100}{\frac{25}{12}} = 100(\frac{12}{25}) = 48$ miles per hour.

NOTE: The harmonic mean h of 2 numbers a and b is found using the formula $h = \frac{2ab}{a+b}$. The harmonic mean is the average rate if the same distance is traveled at two different rates.

If $a = 60$ and $b = 40$, then $h = \frac{2 \cdot 60 \cdot 40}{60+40} = \frac{4800}{100} = 48$ miles per hour.

Difficulty: Easy

NCTM Standard: Algebra Standard for Grades 6–8: explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.

Mathworld.com Classification: NEED CATEGORY!!! DISTANCE-TIME-SPEED

A recipe that makes 5 servings of hot chocolate requires 2 squares of chocolate, $\frac{1}{4}$ cup sugar, 1 cup water and 4 cups milk. Jordan has 5 squares of chocolate, 2 cups of sugar, lots of water and 7 cups of milk. If she maintains the same ratio of ingredients, what is the greatest number of servings of hot chocolate she can make?

(A) $5\dfrac{1}{8}$ **(B)** $6\dfrac{1}{4}$ **(C)** $7\dfrac{1}{2}$ **(D)** $8\dfrac{3}{4}$ **(E)** $9\dfrac{7}{8}$

2009 AMC 8, Problem #15—
"Find out how many servings of hot chocolate can each ingredient make."

Solution

Answer (D): Jordan has 5 squares of chocolate, which is $2\frac{1}{2}$ times the amount the recipe calls for. She has $2 \div \frac{1}{4} = 8$ times the amount of sugar and $\frac{7}{4} = 1\frac{3}{4}$ times the amount of milk necessary to make the recipe. So the amount of milk limits the number of servings. Jordan cannot make more than $5\left(1\frac{3}{4}\right) = 5\left(\frac{7}{4}\right) = \frac{35}{4} = 8\frac{3}{4}$ servings of hot chocolate.

Difficulty: Easy

NCTM Standard: Algebra Standard for Grades 6–8: explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.

Mathworld.com Classification: NEED CATEGORY!!!

How many 3-digit positive integers have digits whose product equals 24?

(A) 12 **(B)** 15 **(C)** 18 **(D)** 21 **(E)** 24

2009 AMC 8, Problem #16—
"First write out the possible ways of expressing 24 as a product of 3 digits."

Solution

Answer (D): The possible ways of expressing 24 as a product of 3 digits are $(1\cdot3\cdot8)$, $(1\cdot4\cdot6)$, $(2\cdot3\cdot4)$ and $(2\cdot2\cdot6)$. From the first product, the six integers 138, 183, 318, 381, 813 and 831 can be formed. Similarly, six integers can be formed from each of the products $(1\cdot4\cdot6)$ and $(2\cdot3\cdot4)$. From the product $(2\cdot2\cdot6)$, the three integers 226, 262 and 622 can be formed. The total number of integers whose digits have a product of 24 is $6+6+6+3=21$.

Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems .

Mathworld.com Classification: Number Theory > Prime Numbers > Prime Factorization > Factor

The positive integers x and y are the two smallest positive integers for which the product of 360 and x is a square and the product of 360 and y is a cube. What is the sum of x and y?

(A) 80 **(B)** 85 **(C)** 115 **(D)** 165 **(E)** 610

2009 AMC 8, Problem #17— "Find the prime factorization of 360."

Solution

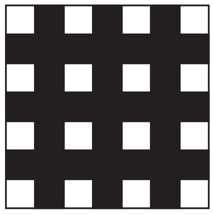
Answer (B): Factor 360 into $2\cdot2\cdot2\cdot3\cdot3\cdot5$. First increase the number of each factor as little as possible to form a square: $2\cdot2\cdot2\cdot2\cdot3\cdot3\cdot5\cdot5 = (2\cdot2\cdot2\cdot3\cdot3\cdot5)(2\cdot5) = (360)(10)$, so x is 10. Then increase the number of each factor as little as possible to form a cube: $2\cdot2\cdot2\cdot3\cdot3\cdot3\cdot5\cdot5\cdot5 = (2\cdot2\cdot2\cdot3\cdot3\cdot5)(3\cdot5\cdot5) = (360)(75)$, so y is 75. The sum of x and y is $10 + 75 = 85$.

Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems .

Mathworld.com Classification: Number Theory > Special Numbers > Figurate Numbers > Square Numbers > Square Number
Number Theory > Special Numbers > Figurate Numbers > Cubic Numbers > Cubic Number

The diagram represents a 7-foot-by-7-foot floor that is tiled with 1-square-foot black tiles and white tiles. Notice that the corners have white tiles. If a 15-foot-by-15-foot floor is to be tiled in the same manner, how many white tiles will be needed?



(A) 49 **(B)** 57 **(C)** 64 **(D)** 96 **(E)** 126

2009 AMC 8, Problem #18—
"To maintain the pattern, white squares will always occupy the corners, and every edge of the square pattern will have an odd number of tiles."

Solution

Answer (C): To maintain the pattern, white squares will always occupy the corners, and every edge of the square pattern will have an odd number of tiles. Create a table, starting with a white square in the corner of the pattern, and increase the sides by 2 tiles.

Floor area	# of white squares	Pattern
1×1	1	1^2
3×3	4	2^2
5×5	9	3^2
7×7	16	4^2
9×9	25	5^2

Following the pattern, an 11×11 area has 36 squares, a 13×13 area has 49, and a 15×15 has 64.

OR

There will be 8 rows that each contain 8 white tiles, so the total is $8(8) = 64$.

Difficulty: Easy

NCTM Standard: Geometry Standard for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: NEED CATEGORY!!! tiles??

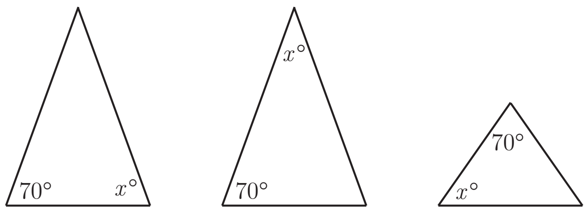
Two angles of an isosceles triangle measure 70° and x°. What is the sum of the three possible values of x?

(A) 95 **(B)** 125 **(C)** 140 **(D)** 165 **(E)** 180

2009 AMC 8, Problem #19— "Sketch the three possible situations."

Solution

Answer (D): The two angles measuring 70° and x°, in an isosceles triangle, could be positioned in three ways, as shown.



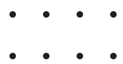
If 70° and x° are the degree measures of the congruent angles, then $x = 70$. If x is the degree measure of the vertex, then x is $180 - 70 - 70 = 40$. If x is the degree measure of one of the base angles, but not 70, then x is $\frac{1}{2}(180 - 70) = 55$.
The possible values of x are 70, 40 and 55. The sum of these values is $70+40+55 = 165$.

Difficulty: Easy

NCTM Standard: Geometry Standard for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Isosceles Triangle

How many non-congruent triangles have vertices at three of the eight points in the array shown below?

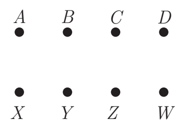


(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2009 AMC 8, Problem #20— “NEED HINT!!!.”

Solution

Answer (D): With the points labeled as shown, one set of non-congruent triangles is AXY, AXZ, AXW, AYZ, AYW, AZW, BXZ and BXW.



Every other possible triangle is congruent to one of the 8 listed triangles.
CHALLENGE: Find the 48 distinct triangles possible and group them into sets of congruent triangles.

Difficulty: Easy

NCTM Standard: Geometry Standard for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Similar Triangles

Andy and Bethany have a rectangular array of numbers with 40 rows and 75 columns. Andy adds the numbers in each row. The average of his 40 sums is A. Bethany adds the numbers in each column. The average of her 75 sums is B. What is the value of $\frac{A}{B}$?

(A) $\dfrac{64}{225}$ **(B)** $\dfrac{8}{15}$ **(C)** 1 **(D)** $\dfrac{15}{8}$ **(E)** $\dfrac{225}{64}$

2009 AMC 8, Problem #21—

"There are 40 rows, so the sum of the 40 row sums is $40A$. This number is also the sum of all of the numbers in the array."

Solution

Answer (D): There are 40 rows, so the sum of the 40 row sums is $40A$. This number is also the sum of all of the numbers in the array because each number in the array is added to obtain one of the row sums. Similarly, there are 75 columns, so the sum of the 75 column sums is $75B$, and this, too, is the sum of all of the numbers in the array. So $40A = 75B$, and $\frac{A}{B} = \frac{75}{40} = \frac{15}{8}$.

Difficulty: Easy

NCTM Standard: Algebra Standard for Grades 6–8: relate and compare different forms of representation for a relationship.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Means > Arithmetic Mean

How many whole numbers between 1 and 1000 do **not** contain the digit 1?

(A) 512 **(B)** 648 **(C)** 720 **(D)** 728 **(E)** 800

2009 AMC 8, Problem #22— "Think of each number between 1 and 1000 as a three-digit number."

Solution

Answer (D): There are 8 one-digit positive integers, excluding 1. There are $8 \cdot 9 = 72$ two-digit integers that do not contain the digit 1. There are $8 \cdot 9 \cdot 9 = 648$ three-digit integers that do not contain the digit 1. There are $8 + 72 + 648 = 728$ integers between 1 and 1000 that do not contain the digit 1.

OR

Think of each number between 1 and 1000 as a three-digit number. For example, think of 2 as 002 and 27 as 027. There are $9^3 = 729$ three-digit numbers that do not use the digit 1. Because 000 does not represent a whole number between 1 and 1000, the total is 728.

Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Number Theory > Arithmetic > Number Bases > Digit

On the last day of school, Mrs. Wonderful gave jelly beans to her class. She gave each boy as many jelly beans as there were boys in the class. She gave each girl as many jelly beans as there were girls in the class. She brought 400 jelly beans, and when she finished, she had six jelly beans left. There were two more boys than girls in her class. How many students were in her class?

(A) 26 **(B)** 28 **(C)** 30 **(D)** 32 **(E)** 34

2009 AMC 8, Problem #23—
"Make a table, starting with a small, reasonable number of girls and boys."

Solution

Answer (B): Mrs. Wonderful gave $400 - 6 = 394$ jelly beans to the class. Make a table, starting with a small, reasonable number of girls and boys.

Girls	Boys	Number of jelly beans
9	11	$(\,9 \times\ 9\,) + (11 \times 11) = 202$
10	12	$(10 \times 10) + (12 \times 12) = 244$
11	13	$(11 \times 11) + (13 \times 13) = 290$
12	14	$(12 \times 12) + (14 \times 14) = 340$
13	15	$(13 \times 13) + (15 \times 15) = 394$

The number of students is $13 + 15 = 28$.

Difficulty: Easy

NCTM Standard: Algebra Standard for Grades 6–8: develop an initial conceptual understanding of different uses of variables.

Mathworld.com Classification: NEED CATEGORY!!!

The letters A, B, C and D all represent different digits. If $\begin{array}{r} A\,B \\ +\,C\,A \\ \hline D\,A \end{array}$ and $\begin{array}{r} A\,B \\ -\,C\,A \\ \hline A \end{array}$, what digit does D represent?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2009 AMC 8, Problem #24— "Because $A + B = A$, $B = 0$."

Solution

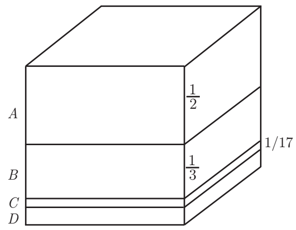
Answer (E): Because $A + B = A$, $B = 0$. Subtracting 10 from A and adding 10 to B, $10 - A = A$, so A must be 5, and AB is 50. Because $50 - C5 = 5$, $C = 4$ and $D = 5 + 4 = 9$.

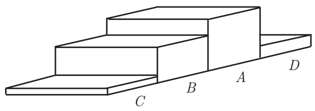
Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.

Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic > Arithmetic

A one-cubic-foot cube is cut into four pieces by three cuts parallel to the top face of the cube. The first cut is $\frac{1}{2}$ foot from the top face. The second cut is $\frac{1}{3}$ foot below the first cut, and the third cut is $\frac{1}{17}$ foot below the second cut. From the top to the bottom the pieces are labeled A, B, C and D. The pieces are then glued together end to end in the order C, B, A, D to make a long solid as shown below. What is the total surface area of this solid in square feet?





(A) 6 **(B)** 7 **(C)** $\frac{419}{51}$ **(D)** $\frac{158}{17}$ **(E)** 11

2009 AMC 8, Problem #25—
"Looking from either end, the visible area totals $\frac{1}{2}$ square foot because piece A measures $\frac{1}{2} \times 1 = \frac{1}{2}$ ft^2."

Solution

Answer (E): Looking from either end, the visible area totals $\frac{1}{2}$ square foot because piece A measures $\frac{1}{2} \times 1 = \frac{1}{2}$ ft^2, and the other pieces decrease in height from that piece. The two side views each show four blocks that can stack to a unit cube. So the area as seen from each side is 1 ft^2. Finally, the top and bottom views each show four unit squares. So the top and bottom view each contribute 4 ft^2 to the area. Summing, the total surface area is

$$\frac{1}{2} + \frac{1}{2} + 1 + 1 + 4 + 4 = 11 \text{ square feet.}$$

CHALLENGE: Suppose the cuts are $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$. Does this change the solution?

Difficulty: Easy

NCTM Standard: Geometry Standard for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Calculus and Analysis > Differential Geometry > Differential Geometry of Surfaces > Surface Area