Susan had $\$50$ to spend at the carnival. She spent $\$12$ on food and twice as much on rides. How many dollars did she have left to spend?

(A) 12 **(B)** 14 **(C)** 26 **(D)** 38 **(E)** 50

2008 AMC 8, Problem #1— "Susan spent $2 \times 12 = \$24$ on rides."

Solution

Answer (B): Susan spent $2 \times 12 = \$24$ on rides, so she had $50 - 12 - 24 = \$14$ to spend.

Difficulty: Easy
NCTM Standard: Number and Operations Standard for Grades 6–8: compute fluently and make reasonable estimates.
Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic > Arithmetic

The ten-letter code BEST OF LUCK represents the ten digits 0–9, in order. What 4-digit number is represented by the code word CLUE?

(A) 8671 **(B)** 8672 **(C)** 9781 **(D)** 9782 **(E)** 9872

2008 AMC 8, Problem #2— "Write out the correspondence between the letters and the numbers."

Solution

Answer (A): Because the key to the code starts with zero, all the letters represent numbers that are one less than their position. Using the key, C is $9 - 1 = 8$, and similarly L is 6, U is 7, and E is 1.
BEST OF LUCK
0123 45 6789
CLUE = 8671

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Recreational Mathematics > Cryptograms > Cipher

If February is a month that contains Friday the 13^{th}, what day of the week is February 1 ?

(A) Sunday **(B)** Monday **(C)** Wednesday **(D)** Thursday **(E)** Saturday

2008 AMC 8, Problem #3— "Counting backward or forward by sevens."

Solution

Answer (A): A week before the 13^{th} is the 6^{th}, which is the first Friday of the month. Counting back from that, the 5^{th} is a Thursday, the 4^{th} is a Wednesday, the 3^{rd} is a Tuesday, the 2^{nd} is a Monday, and the 1^{st} is a Sunday.

OR

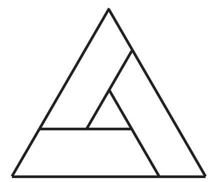
Counting forward by sevens, February 1 occurs on the same day of the week as February 8 and February 15. Because February 13 is a Friday, February 15 is a Sunday, and so is February 1.

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Number Theory > Congruences > Modular Arithmetic

In the figure, the outer equilateral triangle has area 16, the inner equilateral triangle has area 1, and the three trapezoids are congruent. What is the area of one of the trapezoids?



(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

2008 AMC 8, Problem #4—
"The area of the outer triangle with the inner triangle removed is the total area of the three congruent trapezoids."

Solution

Answer (C): The area of the outer triangle with the inner triangle removed is $16-1=15$, the total area of the three congruent trapezoids. Each trapezoid has area $\frac{15}{3}=5$.

Difficulty: Medium-easy

NCTM Standard: Geometry Standard for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Geometric Similarity > Congruent

Barney Schwinn notices that the odometer on his bicycle reads 1441, a palindrome, because it reads the same forward and backward. After riding 4 more hours that day and 6 the next, he notices that the odometer shows another palindrome, 1661. What was his average speed in miles per hour?

(A) 15 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 22

2008 AMC 8, Problem #5—
"The average speed is equal to the total distance divided by the total time."

Solution

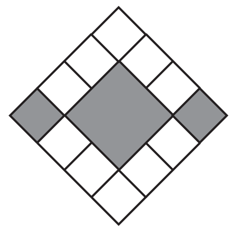
Answer (E): Barney rides $1661 - 1441 = 220$ miles in 10 hours, so his average speed is $\frac{220}{10} = 22$ miles per hour.

Difficulty: Medium-easy

NCTM Standard: Algebra Standard for Grades 6–8: model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Mathworld.com Classification: Algebra > Rate Problems

In the figure, what is the ratio of the area of the gray squares to the area of the white squares?

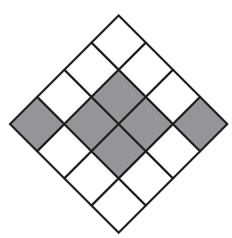


(A) $3:10$ **(B)** $3:8$ **(C)** $3:7$ **(D)** $3:5$ **(E)** $1:1$

2008 AMC 8, Problem #6— "Subdividing the central gray square into unit squares."

Solution

Answer (D): After subdividing the central gray square as shown, 6 of the 16 congruent squares are gray and 10 are white. Therefore, the ratio of the area of the gray squares to the area of the white squares is $6:10$ or $3:5$.



Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Geometry > Plane Geometry > Squares > Square

If $\dfrac{3}{5} = \dfrac{M}{45} = \dfrac{60}{N}$, what is $M + N$?

(A) 27 **(B)** 29 **(C)** 45 **(D)** 105 **(E)** 127

2008 AMC 8, Problem #7— "$M = \frac{3}{5} \cdot 45$ and $N = 60 \cdot \frac{5}{3}$."

Solution

Answer (E): Note that $\frac{M}{45} = \frac{3}{5} = \frac{3 \cdot 9}{5 \cdot 9} = \frac{27}{45}$, so $M = 27$. Similarly, $\frac{60}{N} = \frac{3}{5} = \frac{3 \cdot 20}{5 \cdot 20} = \frac{60}{100}$, so $N = 100$. The sum $M + N = 27 + 100 = 127$.

OR

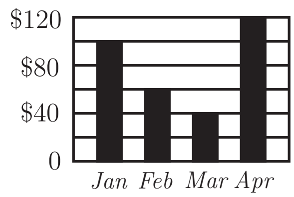
Note that $\frac{M}{45} = \frac{3}{5}$, so $M = \frac{3}{5} \cdot 45 = 27$. Also $\frac{60}{N} = \frac{3}{5}$, so $\frac{N}{60} = \frac{5}{3}$, and $N = \frac{5}{3} \cdot 60 = 100$. The sum $M + N = 27 + 100 = 127$.

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: work flexibly with fractions, decimals, and percents to solve problems.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Ratio

Candy sales of the Boosters Club for January through April are shown. What were the average sales per month in dollars?



(A) 60 **(B)** 70 **(C)** 75 **(D)** 80 **(E)** 85

2008 AMC 8, Problem #8— "Find the total of sales in four months."

Solution

Answer (D): The sales in the 4 months were \$100, \$60, \$40 and \$120. The average sales were $\dfrac{100+60+40+120}{4} = \dfrac{320}{4} = \$80$.

OR

In terms of the \$20 intervals, the sales were 5, 3, 2 and 6 on the chart. Their sum is $5+3+2+6=16$ and the average is $\frac{16}{4} = 4$. The average sales were $4 \cdot \$20 = \$80$.

Difficulty: Easy

NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: select and use appropriate statistical methods to analyze data.

Mathworld.com Classification: Algebra > Rate Problems

In 2005 Tycoon Tammy invested $\$100$ for two years. During the first year her investment suffered a 15% loss, but during the second year the remaining investment showed a 20% gain. Over the two-year period, what was the change in Tammy's investment?

(A) 5% loss **(B)** 2% loss **(C)** 1% gain **(D)** 2% gain **(E)** 5% gain

2008 AMC 8, Problem #9—

"At the end of the first year, Tammy's investment was 85% of the original amount, or $85."

Solution

Answer (D): At the end of the first year, Tammy's investment was 85% of the original amount, or $\$85$. At the end of the second year, she had 120% of her first year's final amount, or 120% of $\$85 = 1.2(\$85) = \$102$. Over the two-year period, Tammy's investment changed from $\$100$ to $\$102$, so she gained 2%.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6–8: compute fluently and make reasonable estimates.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

The average age of the 6 people in Room A is 40. The average age of the 4 people in Room B is 25. If the two groups are combined, what is the average age of all the people?

(A) 32.5 **(B)** 33 **(C)** 33.5 **(D)** 34 **(E)** 35

2008 AMC 8, Problem #10— "Find the sum of the ages of both rooms."

Solution

Answer (D): The sum of the ages of the 6 people in Room A is $6 \times 40 = 240$. The sum of the ages of the 4 people in Room B is $4 \times 25 = 100$. The sum of the ages of the 10 people in the combined group is $100 + 240 = 340$, so the average age of all the people is $\frac{340}{10} = 34$.

Difficulty: Medium-hard

NCTM Standard: Algebra Standard for Grades 6–8: represent and analyze mathematical situations and structures using algebraic symbols.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Means > Arithmetic Mean

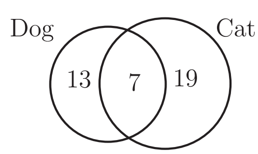
Each of the 39 students in the eighth grade at Lincoln Middle School has one dog or one cat or both a dog and a cat. Twenty students have a dog and 26 students have a cat. How many students have both a dog and a cat?

(A) 7 **(B)** 13 **(C)** 19 **(D)** 39 **(E)** 46

2008 AMC 8, Problem #11— “Use Venn Diagram to represent the situation.”

Solution

Answer (A): Because each student has at least a cat or a dog, there are $39-20=19$ students with a cat but no dog, and $39-26=13$ students with a dog but no cat. So there are $39-13-19=7$ students with both a cat and a dog.

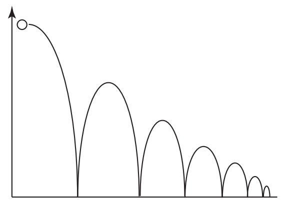


Difficulty: Medium-easy

NCTM Standard: Algebra Standard for Grades 6–8: represent and analyze mathematical situations and structures using algebraic symbols.

Mathworld.com Classification: Foundations of Mathematics > Logic > General Logic > Venn Diagram

A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it first not rise to a height of 0.5 meters?



(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

2008 AMC 8, Problem #12—
"Set up a table to show the height of each bounce."

Solution

Answer (C): The table gives the height of each bounce.

Bounce	1	2	3	4	5
Height in Meters	2	$\frac{2}{3} \cdot 2 =$ $\frac{4}{3}$	$\frac{2}{3} \cdot \frac{4}{3} =$ $\frac{8}{9}$	$\frac{2}{3} \cdot \frac{8}{9} =$ $\frac{16}{27}$	$\frac{2}{3} \cdot \frac{16}{27} =$ $\frac{32}{81}$

Because $\frac{16}{27} > \frac{16}{32} = \frac{1}{2}$ and $\frac{32}{81} < \frac{32}{64} = \frac{1}{2}$, the ball first rises to less than 0.5 meters on the fifth bounce.
Note: Because all the fractions have odd denominators, it is easier to double the numerators than to halve the denominators. So compare $\frac{16}{27}$ and $\frac{32}{81}$ to their numerators' fractional equivalents of $\frac{1}{2}$, $\frac{16}{32}$ and $\frac{32}{64}$.

Difficulty: Medium-hard
NCTM Standard: Algebra Standard for Grades 6–8: model and solve contextualized problems using various representations, such as graphs, tables, and equations.
Mathworld.com Classification: Number Theory > Sequences > Geometric Sequence

Mr. Harman needs to know the combined weight in pounds of three boxes he wants to mail. However, the only available scale is not accurate for weights less than 100 pounds or more than 150 pounds. So the boxes are weighed in pairs in every possible way. The results are 122, 125 and 127 pounds. What is the combined weight in pounds of the three boxes?

(A) 160 **(B)** 170 **(C)** 187 **(D)** 195 **(E)** 354

2008 AMC 8, Problem #13—

"Each box is weighed two times, once with each of the other two boxes."

Solution

Answer (C): Because each box is weighed two times, once with each of the other two boxes, the total $122 + 125 + 127 = 374$ pounds is twice the combined weight of the three boxes. The combined weight is $\frac{374}{2} = 187$ pounds.

Difficulty: Medium

NCTM Standard: Algebra Standard for Grades 6–8: use mathematical models to represent and understand quantitative relationships.

Mathworld.com Classification: Discrete Mathematics > Combinatorics > Weighing > Weighing

Three As, three Bs and three Cs are placed in the nine spaces so that each row and column contain one of each letter. If A is placed in the upper left corner, how many arrangements are possible?

A		

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

2008 AMC 8, Problem #14—
"There are only two possible spaces for the B in row 1 and only two possible spaces for the A in row 2."

Solution

Answer (C): There are only two possible spaces for the B in row 1 and only two possible spaces for the A in row 2. Once these are placed, the entries in the remaining spaces are determined.
The four arrangements are:

A	B	C
B	C	A
C	A	B

A	B	C
C	A	B
B	C	A

A	C	B
C	B	A
B	A	C

A	C	B
B	A	C
C	B	A

OR

The As can be placed either

A		
	A	
		A

or

A		
		A
	A	

In each case, the letter next to the top A can be B or C. At that point the rest of the grid is completely determined. So there are $2 + 2 = 4$ possible arrangements.

Difficulty: Medium-hard

NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: develop and evaluate inferences and predictions that are based on data.

Mathworld.com Classification: Discrete Mathematics > Combinatorics > Designs > Latin Square

In Theresa's first 8 basketball games, she scored 7, 4, 3, 6, 8, 3, 1 and 5 points. In her ninth game, she scored fewer than 10 points and her points-per-game average for the nine games was an integer. Similarly in her tenth game, she scored fewer than 10 points and her points-per-game average for the 10 games was also an integer. What is the product of the number of points she scored in the ninth and tenth games?

(A) 35 **(B)** 40 **(C)** 48 **(D)** 56 **(E)** 72

2008 AMC 8, Problem #15—

"The sum of the points Theresa scored in the first 8 games is 37. After the ninth game, her point total must be a multiple of 9."

Solution

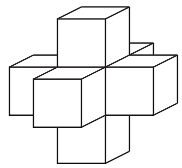
Answer (B): The sum of the points Theresa scored in the first 8 games is 37. After the ninth game, her point total must be a multiple of 9 between 37 and $37 + 9 = 46$, inclusive. The only such point total is $45 = 37 + 8$, so in the ninth game she scored 8 points. Similarly, the next point total must be a multiple of 10 between 45 and $45 + 9 = 54$. The only such point total is $50 = 45 + 5$, so in the tenth game she scored 5 points. The product of the number of points scored in Theresa's ninth and tenth games is $8 \cdot 5 = 40$.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Means > Arithmetic Mean

A shape is created by joining seven unit cubes, as shown. What is the ratio of the volume in cubic units to the surface area in square units?



(A) $1:6$ **(B)** $7:36$ **(C)** $1:5$ **(D)** $7:30$ **(E)** $6:25$

2008 AMC 8, Problem #16—
"Six of the cubes have 5 square faces exposed. The middle cube has no face exposed."

Solution

Answer (D): The volume is $7 \times 1 = 7$ cubic units. Six of the cubes have 5 square faces exposed. The middle cube has no face exposed. So the total surface area of the figure is $5 \times 6 = 30$ square units. The ratio of the volume to the surface area is $7:30$.

OR

The volume is $7 \times 1 = 7$ cubic units. There are five unit squares facing each of six directions: front, back, top, bottom, left and right, for a total of 30 square units of surface area. The ratio of the volume to the surface area is $7:30$.

Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Solid Geometry > Polyhedra > Cubes > Unit Cube

Ms. Osborne asks each student in her class to draw a rectangle with integer side lengths and a perimeter of 50 units. All of her students calculate the area of the rectangle they draw. What is the difference between the largest and smallest possible areas of the rectangles?

(A) 76 **(B)** 120 **(C)** 128 **(D)** 132 **(E)** 136

2008 AMC 8, Problem #17— "Make a table with all possible combination of lengths and widths."

Solution

Answer (D): The formula for the perimeter of a rectangle is $2l + 2w$, so $2l + 2w = 50$, and $l + w = 25$. Make a chart of the possible widths, lengths, and areas, assuming all the widths are shorter than all the lengths.

Width	1	2	3	4	5	6	7	8	9	10	11	12
Length	24	23	22	21	20	19	18	17	16	15	14	13
Area	24	46	66	84	100	114	126	136	144	150	154	156

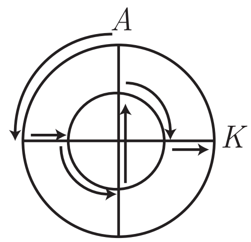
The largest possible area is $13 \times 12 = 156$ and the smallest is $1 \times 24 = 24$, for a difference of $156 - 24 = 132$ square units.
Note: The product of two numbers with a fixed sum increases as the numbers get closer together. That means, given the same perimeter, the square has a larger area than any rectangle, and a rectangle with a shape closest to a square will have a larger area than other rectangles with equal perimeters.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Geometry > Plane Geometry > Rectangles > Rectangle

Two circles that share the same center have radii 10 meters and 20 meters. An aardvark runs along the path shown, starting at A and ending at K. How many meters does the aardvark run?



(A) $10\pi + 20$ **(B)** $10\pi + 30$ **(C)** $10\pi + 40$ **(D)** $20\pi + 20$ **(E)** $20\pi + 40$

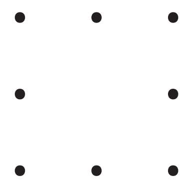
2008 AMC 8, Problem #18— "Calculate the legs of the aardvark's trip individually."

Solution

Answer (E): The length of first leg of the aardvark's trip is $\frac{1}{4}(2\pi \times 20) = 10\pi$ meters. The third and fifth legs are each $\frac{1}{4}(2\pi \times 10) = 5\pi$ meters long. The second and sixth legs are each 10 meters long, and the length of the fourth leg is 20 meters. The length of the total trip is $10\pi + 5\pi + 5\pi + 10 + 10 + 20 = 20\pi + 40$ meters.

Difficulty: Medium-hard
NCTM Standard: Geometry Standard for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
Mathworld.com Classification: Geometry > Plane Geometry > Circles > Circle

Eight points are spaced at intervals of one unit around a 2×2 square, as shown. Two of the 8 points are chosen at random. What is the probability that the points are one unit apart?

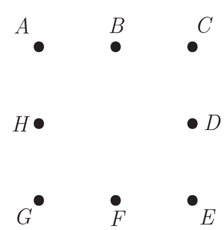


(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{4}{11}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{4}{7}$

2008 AMC 8, Problem #19—
"There are totally 28 ways to pick the pair of points."

Solution

Answer (B): Choose two points. Any of the 8 points can be the first choice, and any of the 7 other points can be the second choice. So there are $8 \times 7 = 56$ ways of choosing the points in order. But each pair of points is counted twice, so there are $\frac{56}{2} = 28$ possible pairs.



Label the eight points as shown. Only segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$ and $\overline{HA}$ are 1 unit long. So 8 of the 28 possible segments are 1 unit long, and the probability that the points are one unit apart is $\frac{8}{28} = \frac{2}{7}$.

OR

Pick the two points, one at a time. No matter how the first point is chosen, exactly 2 of the remaining 7 points are 1 unit from this point. So the probability of the second point being 1 unit from the first is $\frac{2}{7}$.

Difficulty: Medium-hard

NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: understand and apply basic concepts of probability.

Mathworld.com Classification: Probability and Statistics > Probability > Probability

The students in Mr. Neatkin's class took a penmanship test. Two-thirds of the boys and $\frac{3}{4}$ of the girls passed the test, and an equal number of boys and girls passed the test. What is the minimum possible number of students in the class?

(A) 12 **(B)** 17 **(C)** 24 **(D)** 27 **(E)** 36

2008 AMC 8, Problem #20—

"The number of boys in the class is a multiple of 3, and the number of girls in the class is a multiple of 4."

Solution

Answer (B): Because $\frac{2}{3}$ of the boys passed, the number of boys in the class is a multiple of 3. Because $\frac{3}{4}$ of the girls passed, the number of girls in the class is a multiple of 4. Set up a chart and compare the number of boys who passed with the number of girls who passed to find when they are equal.

Total boys	Boys passed
3	2
6	4
9	6

Total girls	Girls passed
4	3
8	6

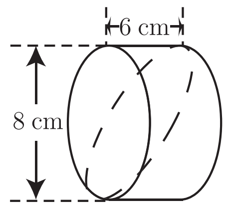
The first time the number of boys who passed equals the number of girls who passed is when they are both 6. The minimum possible number of students is $9 + 8 = 17$.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6–8: understand and use ratios and proportions to represent quantitative relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Fraction

Jerry cuts a wedge from a 6-cm cylinder of bologna as shown by the dashed curve. Which answer choice is closest to the volume of his wedge in cubic centimeters?



(A) 48 **(B)** 75 **(C)** 151 **(D)** 192 **(E)** 603

2008 AMC 8, Problem #21—
"The formula for the volume of a cylinder is $V = \pi r^2 h$."

Solution

Answer (C): Using the formula for the volume of a cylinder, the bologna has volume $\pi r^2 h = \pi \times 4^2 \times 6 = 96\pi$. The cut divides the bologna in half. The half-cylinder will have volume $\frac{96\pi}{2} = 48\pi \approx 151$ cm^3.
Note: The value of π is slightly greater than 3, so to estimate the volume multiply $48(3) = 144$ cm^3. The product is slightly less than and closer to answer C than any other answer.

Difficulty: Medium-hard
NCTM Standard: Geometry Standard for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
Mathworld.com Classification: Geometry > Solid Geometry > Cylinders > Cylinder

For how many positive integer values of n are both $\frac{n}{3}$ and $3n$ three-digit whole numbers?

(A) 12 **(B)** 21 **(C)** 27 **(D)** 33 **(E)** 34

2008 AMC 8, Problem #22— "Since $\frac{n}{3}$ and $3n$ are three-digit numbers, $n \in [300, 333]$."

Solution

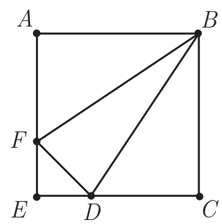
Answer (A): Because $\frac{n}{3}$ is at least 100 and is an integer, n is at least 300 and is a multiple of 3. Because $3n$ is at most 999, n is at most 333. The possible values of n are 300, 303, 306, …, $333 = 3 \cdot 100$, $3 \cdot 101$, $3 \cdot 102$, …, $3 \cdot 111$, so the number of possible values is $111 - 100 + 1 = 12$.

Difficulty: Medium-hard

NCTM Standard: Algebra Standard for Grades 6–8: represent and analyze mathematical situations and structures using algebraic symbols.

Mathworld.com Classification: Number Theory > Integers > Whole Number

In square $ABCE$, $AF = 2FE$ and $CD = 2DE$. What is the ratio of the area of $\triangle BFD$ to the area of square $ABCE$?

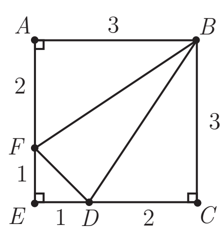


(A) $\frac{1}{6}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{5}{18}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{7}{20}$

2008 AMC 8, Problem #23—
"Area of $\triangle BFD$ equals the area of $ABCD$ minus sum of areas of $\triangle ABF$, $\triangle FED$ and $\triangle BCD$."

Solution

Answer (C): Because the answer is a ratio, it does not depend on the side length of the square. Let $AF = 2$ and $FE = 1$. That means square $ABCE$ has side length 3 and area $3^2 = 9$ square units. The area of $\triangle BAF$ is equal to the area of $\triangle BCD = \frac{1}{2} \cdot 3 \cdot 2 = 3$ square units. Triangle DEF is an isosceles right triangle with leg lengths $DE = FE = 1$. The area of $\triangle DEF$ is $\frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$ square units. The area of $\triangle BFD$ is equal to the area of the square minus the areas of the three right triangles: $9 - (3 + 3 + \frac{1}{2}) = \frac{5}{2}$. So the ratio of the area of $\triangle BFD$ to the area of square $ABCE$ is $\frac{\frac{5}{2}}{9} = \frac{5}{18}$.



Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

Ten tiles numbered 1 through 10 are turned face down. One tile is turned up at random, and a die is rolled. What is the probability that the product of the numbers on the tile and the die will be a square?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{11}{60}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{7}{30}$

2008 AMC 8, Problem #24—
"There are $10 \times 6 = 60$ possible pairs."

Solution

Answer (C): There are $10 \times 6 = 60$ possible pairs. The squares less than 60 are 1, 4, 9, 16, 25, 36 and 49. The possible pairs with products equal to the given squares are $(1,1)$, $(2,2)$, $(1,4)$, $(4,1)$, $(3,3)$, $(9,1)$, $(4,4)$, $(8,2)$, $(5,5)$, $(6,6)$ and $(9,4)$. So the probability is $\frac{11}{60}$.

Difficulty: Medium-hard

NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.

Mathworld.com Classification: Probability and Statistics > Probability > Probability

Margie's winning art design is shown. The smallest circle has radius 2 inches, with each successive circle's radius increasing by 2 inches. Approximately what percent of the design is black?

(A) 42 **(B)** 44 **(C)** 45 **(D)** 46 **(E)** 48

2008 AMC 8, Problem #25— "Calculate the areas of all circles."

Solution

Answer (A):

Circle #	Radius	Area
1	2	4π
2	4	16π
3	6	36π
4	8	64π
5	10	100π
6	12	144π

The total black area is $4\pi + (36-16)\pi + (100-64)\pi = 60\pi$ in^2.
So the percent of the design that is black is $100 \times \frac{60\pi}{144\pi} = 100 \times \frac{5}{12}$ or about 42%.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Circles > Circle