Theresa's parents have agreed to buy her tickets to see her favorite band if she spends an average of 10 hours per week helping around the house for 6 weeks. For the first 5 weeks she helps around the house for 8, 11, 7, 12 and 10 hours. How many hours must she work during the final week to earn the tickets?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

2007 AMC 8, Problem #1—
"Theresa needs to help around the house for a total of $10 \times 6 = 60$ **hours."**

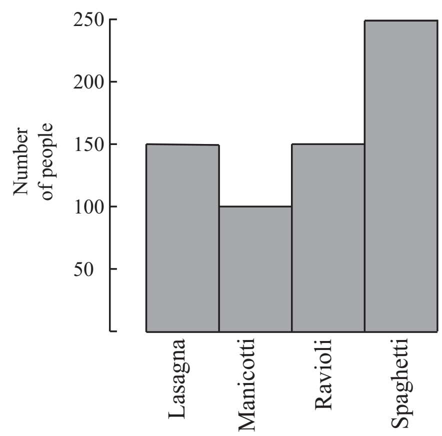
Solution (D) The first 5 weeks Theresa works a total of $8 + 11 + 7 + 12 + 10 = 48$ hours. She has promised to work $6 \times 10 = 60$ hours. She must work $60 - 48 = 12$ hours during the final week.

Difficulty: Easy

NCTM Standard: Algebra Standard for Grades 6–8: use mathematical models to represent and understand quantitative relationships.

Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic > Arithmetic

Six-hundred fifty students were surveyed about their pasta preferences. The choices were lasagna, manicotti, ravioli and spaghetti. The results of the survey are displayed in the bar graph. What is the ratio of the number of students who preferred spaghetti to the number of students who preferred manicotti?



(A) $\frac{2}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{4}$ **(D)** $\frac{5}{3}$ **(E)** $\frac{5}{2}$

2007 AMC 8, Problem #2—
"Represent quantitative relationships with ratios."

Solution (E) The ratio of the number of students who preferred spaghetti to the number of students who preferred manicotti is $\frac{250}{100} = \frac{5}{2}$.

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand and use ratios and proportions to represent quantitative relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Ratio

What is the sum of the two smallest prime factors of 250?

(A) 2 **(B)** 5 **(C)** 7 **(D)** 10 **(E)** 12

2007 AMC 8, Problem #3—
"Write out the prime factorization of 250."

Solution (C) The prime factorization of 250 is $2 \cdot 5 \cdot 5 \cdot 5$. The sum of 2 and 5 is 7.

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: use factors, multiples, prime factorization, and relatively prime numbers to solve problems.

Mathworld.com Classification: Number Theory > Prime Numbers > Prime Factorization

A haunted house has six windows. In how many ways can Georgie the Ghost enter the house by one window and leave by a different window?

(A) 12 **(B)** 15 **(C)** 18 **(D)** 30 **(E)** 36

2007 AMC 8, Problem #4—

"After Georgie picks the first window, how many choices does he have for picking the second window?"

Solution (D) Georgie has 6 choices for the window in which to enter. After entering, Georgie has 5 choices for the window from which to exit. So altogether there are $6 \times 5 = 30$ different ways for Georgie to enter one window and exit another.

Difficulty: Medium

NCTM Standard: Algebra Standard for Grades 6–8: use mathematical models to represent and understand quantitative relationships.

Mathworld.com Classification: Discrete Mathematics > Combinatorics > General Combinatorics > Counting Generalized Principle

Chandler wants to buy a $\$500$ mountain bike. For his birthday, his grandparents send him $\$50$, his aunt sends him $\$35$ and his cousin gives him $\$15$. He earns $\$16$ per week for his paper route. He will use all of his birthday money and all of the money he earns from his paper route. In how many weeks will he be able to buy the mountain bike?

(A) 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

2007 AMC 8, Problem #5—
"How many dollars does Chandler have to earn from his paper route?"

Solution (B) For his birthday, Chandler gets $50 + 35 + 15 = 100$ dollars. Therefore, he needs $500 - 100 = 400$ dollars more. It will take Chandler $400 \div 16 = 25$ weeks to earn 400 dollars, so he can buy his bike after 25 weeks.

Difficulty: Easy
NCTM Standard: Problem Solving for Grades 6–8: solve problems that arise in mathematics and in other contexts.
Mathworld.com Classification: Number Theory > Arithmetic > General Arithmetic > Arithmetic

The average cost of a long-distance call in the USA in 1985 was 41 cents per minute, and the average cost of a long-distance call in the USA in 2005 was 7 cents per minute. Find the approximate percent decrease in the cost per minute of a long-distance call.

(A) 7 **(B)** 17 **(C)** 34 **(D)** 41 **(E)** 80

2007 AMC 8, Problem #6—
"Percentage decreased $= \frac{\text{price difference}}{\text{old price}}$."

Solution (E) The difference in the cost of a long-distance call per minute from 1985 to 2005 was $41 - 7 = 34$ cents. The percent decrease is $100 \times \frac{34}{41} \approx 100 \times \frac{32}{40} = 100 \times \frac{8}{10} = 80\%$.

Difficulty: Medium-hard

NCTM Standard: Number and Operations for Grades 6–8: work flexibly with fractions, decimals, and percents to solve problems.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

The average age of 5 people in a room is 30 years. An 18-year-old person leaves the room. What is the average age of the four remaining people?

(A) 25 **(B)** 26 **(C)** 29 **(D)** 33 **(E)** 36

2007 AMC 8, Problem #7—
"What is the sum of the ages of the people in the room originally?"

Solution (D) Originally the sum of the ages of the people in the room is $5 \times 30 = 150$. After the 18-year-old leaves, the sum of the ages of the remaining people is $150 - 18 = 132$. So the average age of the four remaining people is $\frac{132}{4} = 33$ years.

OR

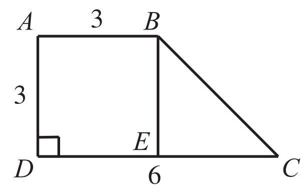
The 18-year-old is 12 years younger than 30, so the four remaining people are an average of $\frac{12}{4} = 3$ years older than 30.

Difficulty: Medium

NCTM Standard: Number and Operations for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Means > Arithmetic Mean

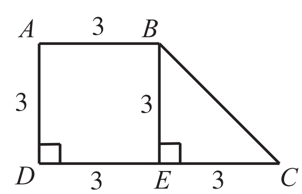
In trapezoid $ABCD$, $\overline{AD}$ is perpendicular to $\overline{DC}$, $AD = AB = 3$, and $DC = 6$. In addition, E is on $\overline{DC}$, and $\overline{BE}$ is parallel to $\overline{AD}$. Find the area of $\triangle BEC$.



(A) 3 **(B)** 4.5 **(C)** 6 **(D)** 9 **(E)** 18

2007 AMC 8, Problem #8— "Triangle BEC is a right triangle."

Solution (B) Note that $ABED$ is a square with side 3. Subtract DE from DC, to find that $\overline{EC}$, the base of $\triangle BEC$, has length 3. The area of $\triangle BEC$ is $\frac{1}{2} \cdot 3 \cdot 3 = \frac{9}{2} = 4.5$.



OR

The area of the $\triangle BEC$ is the area of the trapezoid $ABCD$ minus the area of the square $ABED$. The area of $\triangle BEC$ is $\frac{1}{2}(3+6)3 - 3^2 = 13.5 - 9 = 4.5$.

Difficulty: Medium

NCTM Standard: Geometry for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Geometry > Plane Geometry > Quadrilaterals > Trapezoid

To complete the grid below, each of the digits 1 through 4 must occur once in each row and once in each column. What number will occupy the lower right-hand square?

1		2	
2	3		
			4

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** cannot be determined

2007 AMC 8, Problem #9— "The number in the last column of the second row must be 1."

Solution (B) The number in the last column of the second row must be 1 because there are already a 2 and a 3 in the second row and a 4 in the last column. By similar reasoning, the number above the 1 must be 3. So the number in the lower right-hand square must be 2. This is not the only way to find the solution.

1		2	3
2	3		1
			4
			2

The completed square is

1	4	2	3
2	3	4	1
3	2	1	4
4	1	3	2

Difficulty: Medium-easy

NCTM Standard: Number and Operations for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Recreational Mathematics > Mathematical Records > Latin Square

For any positive integer n, define $\boxed{n}$ to be the sum of the positive factors of n. For example, $\boxed{6} = 1 + 2 + 3 + 6 = 12$. Find $\boxed{\boxed{11}}$.

(A) 13 **(B)** 20 **(C)** 24 **(D)** 28 **(E)** 30

2007 AMC 8, Problem #10—
"$\boxed{\mathbf{11}} = 1 + 11 = 12$."

Solution (D) First calculate $\boxed{11} = 1 + 11 = 12$. So

$$\boxed{\boxed{11}} = \boxed{12} = 1 + 2 + 3 + 4 + 6 + 12 = 28$$

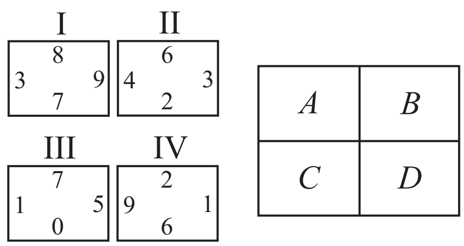
.

Difficulty: Medium-hard

NCTM Standard: Number and Operations for Grades 6–8: understand meanings of operations and how they relate to one another.

Mathworld.com Classification: Calculus and Analysis > Functions > Unary Operation

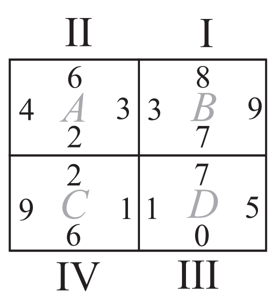
Tiles I, II, III and IV are translated so one tile coincides with each of the rectangles A, B, C and D. In the final arrangement, the two numbers on any side common to two adjacent tiles must be the same. Which of the tiles is translated to Rectangle C?



(A) I **(B)** II **(C)** III **(D)** IV **(E)** cannot be determined

2007 AMC 8, Problem #11—
"Because Tile III has a 0 on the bottom edge and there is no 0 on any other tile, Tile III must be placed on C or D."

Solution (D) Because Tile III has a 0 on the bottom edge and there is no 0 on any other tile, Tile III must be placed on C or D. Because Tile III has a 5 on the right edge and there is no 5 on any other tile, Tile III must be placed on the right, on D. Because Tile III has a 1 on the left edge and only Tile IV has a 1 on the right edge, Tile IV must be placed to the left of Tile III, that is, on C .

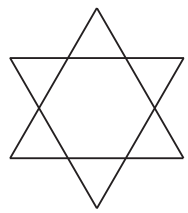


Difficulty: Medium-hard

NCTM Standard: Geometry for Grades 6–8: examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

Mathworld.com Classification: Geometry > Plane Geometry > Tiling > Tessellation

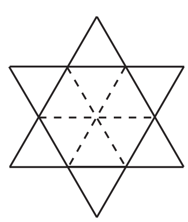
A unit hexagram is composed of a regular hexagon of side length 1 and its 6 equilateral triangular extensions, as shown in the diagram. What is the ratio of the area of the extensions to the area of the original hexagon?



(A) $1:1$ **(B)** $6:5$ **(C)** $3:2$ **(D)** $2:1$ **(E)** $3:1$

2007 AMC 8, Problem #12—
"Use diagonals to cut the hexagon into 6 congruent triangles."

Solution (A) Use diagonals to cut the hexagon into 6 congruent triangles. Because each exterior triangle is also equilateral and shares an edge with an internal triangle, each exterior triangle is congruent to each interior triangle. Therefore, the ratio of the area of the extensions to the area of the hexagon is 1:1.

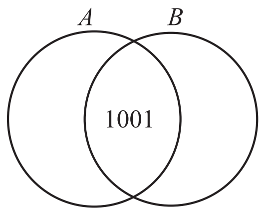


Difficulty: Medium

NCTM Standard: Geometry for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Polygons > Hexagram

Sets A and B, shown in the Venn diagram, have the same number of elements. Their union has 2007 elements and their intersection has 1001 elements. Find the number of elements in A.



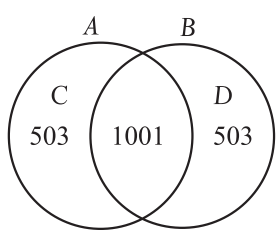
(A) 503 **(B)** 1006 **(C)** 1504 **(D)** 1507 **(E)** 1510

2007 AMC 8, Problem #13—
"The sum of elements in A and B is $2007 + 1001 = 3008$."

Solution (C) Let C denote the set of elements that are in A but not in B. Let D denote the set of elements that are in B but not in A. Because sets A and B have the same number of elements, the number of elements in C is the same as the number of elements in D. This number is half the number of elements in the union of A and B minus the intersection of A and B. That is, the number of elements in each of C and D is

$$\frac{1}{2}(2007 - 1001) = \frac{1}{2} \cdot 1006 = 503.$$

Adding the number of elements in A and B to the number in A but not in B gives $1001 + 503 = 1504$ elements in A.



OR

Let x be the number of elements each in A and B. Then $2x - 1001 = 2007$, $2x = 3008$ and $x = 1504$.

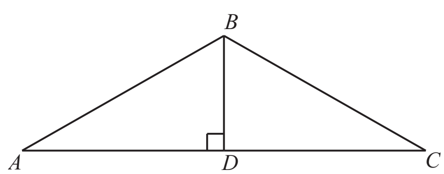
Difficulty: Hard
NCTM Standard: Number and Operations for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.
Mathworld.com Classification: Foundations of Mathematics > Logic > General Logic > Venn Diagram

The base of isosceles $\triangle ABC$ is 24 and its area is 60. What is the length of one of the congruent sides?

(A) 5 **(B)** 8 **(C)** 13 **(D)** 14 **(E)** 18

2007 AMC 8, Problem #14—
"Draw $\overline{BD}$ to be the altitude from B to $\overline{AC}$."

Solution (C) Let $\overline{BD}$ be the altitude from B to $\overline{AC}$ in $\triangle ABC$.



Then $60 =$ the area of $\triangle ABC = \frac{1}{2} \cdot 24 \cdot BD$, so $BD = 5$. Because $\triangle ABC$ is isosceles, $\triangle ABD$ and $\triangle CBD$ are congruent right triangles. This means that $AD = DC = \dfrac{24}{2} = 12$. Applying the Pythagorean Theorem to $\triangle ABD$ gives

$$AB^2 = 5^2 + 12^2 = 169 = 13^2, \text{ so } AB = 13.$$

Difficulty: Hard

NCTM Standard: Geometry for Grades 6–8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Isosceles Triangle

Let a, b and c be numbers with $0 < a < b < c$. Which of the following is impossible?

(A) $a + c < b$ **(B)** $a \cdot b < c$ **(C)** $a + b < c$ **(D)** $a \cdot c < b$ **(E)** $\frac{b}{c} = a$

2007 AMC 8, Problem #15—
"Note that a, b, c are all positive numbers."

Solution (A) Because $b < c$ and $0 < a$, adding corresponding sides of the inequalities gives $b < a+c$, so (A) is impossible. To see that the other choices are possible, consider the following choices for a, b, and c:

(B) and (C): $a = 1$, $b = 2$, and $c = 4$;

(D): $a = \frac{1}{3}$, $b = 1$, and $c = 2$;

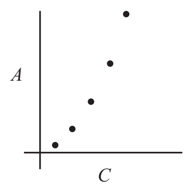
(E): $a = \frac{1}{2}$, $b = 1$, and $c = 2$.

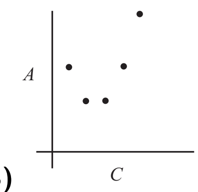
Difficulty: Medium

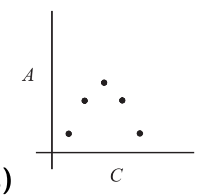
NCTM Standard: Number and Operations for Grades 6–8: understand meanings of operations and how they relate to one another.

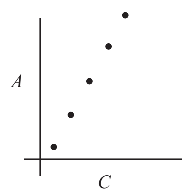
Mathworld.com Classification: Calculus and Analysis > Inequalities > Inequality

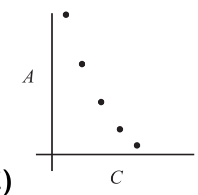
Amanda Reckonwith draws five circles with radii 1, 2, 3, 4 and 5. Then for each circle she plots the point (C, A), where C is its circumference and A is its area. Which of the following could be her graph?

(A)


(B)


(C)


(D)


(E)


2007 AMC 8, Problem #16—
"Find the circumferences and areas for the five circles."

Solution (A) The circumferences of circles with radii 1 through 5 are 2π, 4π, 6π, 8π and 10π, respectively. Their areas are, respectively, $\pi, 4\pi$, 9π, 16π and 25π. The points $(2\pi, \pi)$, $(4\pi, 4\pi)$, $(6\pi, 9\pi)$, $(8\pi, 16\pi)$ and $(10\pi, 25\pi)$ are graphed in **(A)**. It is the only graph of an increasing quadratic function, called a parabola .

Difficulty: Medium-hard

NCTM Standard: Geometry for Grades 6–8: specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Mathworld.com Classification: Geometry > Plane Geometry > Circles > Circle

A mixture of 30 liters of paint is 25% red tint, 30% yellow tint and 45% water. Five liters of yellow tint are added to the original mixture. What is the percent of yellow tint in the new mixture?

(A) 25 **(B)** 35 **(C)** 40 **(D)** 45 **(E)** 50

2007 AMC 8, Problem #17—
"There are $0.30(30) = 9$ liters of yellow tint in the original 30-liter mixture."

Solution (C) There are $0.30(30) = 9$ liters of yellow tint in the original 30-liter mixture. After adding 5 liters of yellow tint, 14 of the 35 liters of the new mixture are yellow tint. The percent of yellow tint in the new mixture is $100 \times \frac{14}{35} = 100 \times \frac{2}{5}$ or 40%.

Difficulty: Medium-hard

NCTM Standard: Number and Operations for Grades 6–8: work flexibly with fractions, decimals, and percents to solve problems.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

The product of the two 99-digit numbers

$$303{,}030{,}303,\ldots,030{,}303 \quad \text{and} \quad 505{,}050{,}505,\ldots,050{,}505$$

has thousands digit A and units digit B. What is the sum of A and B?

(A) 3 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

2007 AMC 8, Problem #18—

"To find A and B, it is sufficient to consider only $303 \cdot 505$, because 0 is in the thousands place in both factors."

Solution (D) To find A and B, it is sufficient to consider only $303 \cdot 505$, because 0 is in the thousands place in both factors.

$$\begin{array}{r} \cdots 303 \\ \times \quad \cdots 505 \\ \hline \cdots 1515 \\ \cdots 1500 \\ \hline \cdots 3015 \end{array}$$

So $A = 3$ and $B = 5$, and the sum is $A + B = 3 + 5 = 8$.

Difficulty: Medium

NCTM Standard: Number and Operations for Grades 6–8: understand meanings of operations and how they relate to one another.

Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division > Multiplication

Pick two consecutive positive integers whose sum is less than 100. Square both of those integers and then find the difference of the squares. Which of the following could be the difference?

(A) 2 **(B)** 64 **(C)** 79 **(D)** 96 **(E)** 131

2007 AMC 8, Problem #19—
"One of the squares of two consecutive integers is odd and the other is even, so their difference must be odd."

Solution (C) One of the squares of two consecutive integers is odd and the other is even, so their difference must be odd. This eliminates A, B and D. The largest consecutive integers that have a sum less than 100 are 49 and 50, whose squares are 2401 and 2500, with a difference of 99. Because the difference of the squares of consecutive positive integers increases as the integers increase, the difference cannot be 131. The difference between the squares of 40 and 39 is 79.

OR

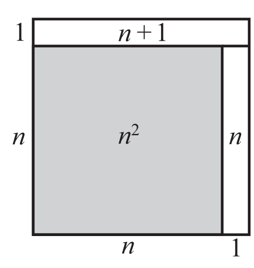
Let the consecutive integers be n and $n+1$, with $n \leq 49$. Then

$$(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1 = n + (n + 1).$$

That means the difference of the squares is an odd number. Therefore, the difference is an odd number less than or equal to $49 + (49 + 1) = 99$, and choice C is the only possible answer. The sum of $n = 39$ and $n + 1 = 40$ is 79.

Note: The difference of the squares of any two consecutive integers is not only odd but also the sum of the two consecutive integers. Every positive odd integer greater than 1 and less than 100 could be the answer.

Seen in geometric terms, $(n+1)^2 - n^2$ looks like



Difficulty: Medium-hard

NCTM Standard: Number and Operations for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Number Theory > Special Numbers > Figurate Numbers > Square Numbers > Square

Before district play, the Unicorns had won 45% of their basketball games. During district play, they won six more games and lost two, to finish the season having won half their games. How many games did the Unicorns play in all?

(A) 48 **(B)** 50 **(C)** 52 **(D)** 54 **(E)** 60

2007 AMC 8, Problem #20—
"Won half games $= 50\% = \frac{45\% \cdot \textbf{non-district games} + 6}{\textbf{non-district games} + 6 + 2}$ **."**

Solution (A) Because 45% is the same as the simplified fraction $\frac{9}{20}$, the Unicorns won 9 games for each 20 games they played. This means that the Unicorns must have played some multiple of 20 games before district play. The table shows the possibilities that satisfy the conditions in the problem.

Before District Play			**After District Play**		
Games Played	Games Won	Games Lost	Games Played	Games Won	Games Lost
20	9	11	28	15	13
40	**18**	**22**	**48**	**24**	**24**
60	27	33	68	33	35
80	36	44	88	42	46
...	...	...	...	...	...

Only when the Unicorns played 40 games before district play do they finish winning half of their games. So the Unicorns played $24 + 24 = 48$ games.

OR

Let n be the number of Unicorn games before district play. Then $0.45n+6 = 0.5(n+8)$. Solving for n yields

$$\begin{aligned} 0.45n + 6 &= 0.5n + 4, \\ 2 &= 0.05n, \\ 40 &= n. \end{aligned}$$

So the total number of games is $40 + 8 = 48$.

Difficulty: Medium-hard

NCTM Standard: Algebra for Grades 6–8: represent and analyze mathematical situations and structures using algebraic symbols.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

Two cards are dealt from a deck of four red cards labeled A, B, C, D and four green cards labeled A, B, C, D. A winning pair is two of the same color or two of the same letter. What is the probability of drawing a winning pair?

(A) $\frac{2}{7}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{5}{8}$

2007 AMC 8, Problem #21—
"After the first card is dealt, there are seven left. How many of the remaining cards are winners?"

Solution (D) After the first card is dealt, there are seven left. The three cards with the same color as the initial card are winners and so is the card with the same letter but a different color. That means four of the remaining seven cards form winning pairs with the first card, so the probability of winning is $\frac{4}{7}$.

Difficulty: Hard
NCTM Standard: Probability for Grades 6–8: understand and apply basic concepts of probability.

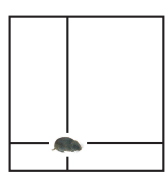
Mathworld.com Classification: Probability and Statistics > Probability > Probability

A lemming sits at a corner of a square with side length 10 meters. The lemming runs 6.2 meters along a diagonal toward the opposite corner. It stops, makes a 90° right turn and runs 2 more meters.
A scientist measures the shortest distance between the lemming and each side of the square. What is the average of these four distances in meters?

(A) 2 **(B)** 4.5 **(C)** 5 **(D)** 6.2 **(E)** 7

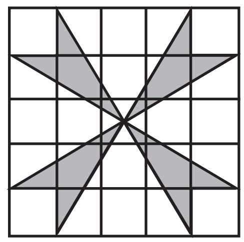
2007 AMC 8, Problem #22—
"Wherever the lemming is inside the square, the sum of the distances to the two horizontal sides is 10 meters and the sum of the distances to the two vertical sides is 10 meters."

Solution (C) Wherever the lemming is inside the square, the sum of the distances to the two horizontal sides is 10 meters and the sum of the distances to the two vertical sides is 10 meters. Therefore the sum of all four distances is 20 meters, and the average of the four distances is $\frac{20}{4} = 5$ meters.



Difficulty: Medium-hard
NCTM Standard: Geometry for Grades 6–8: use visualization, spatial reasoning, and geometric modeling to solve problems.
Mathworld.com Classification: Geometry > Plane Geometry > Squares > Square

What is the area of the shaded pinwheel shown in the 5×5 grid?



(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

2007 AMC 8, Problem #23— "Find the area of the unshaded portion."

Solution (B) Find the area of the unshaded portion of the 5×5 grid, then subtract the unshaded area from the total area of the grid. The unshaded triangle in the middle of the top of the 5×5 grid has a base of 3 and an altitude of $\frac{5}{2}$. The four unshaded triangles have a total area of $4 \times \frac{1}{2} \times 3 \times \frac{5}{2} = 15$ square units. The four corner squares are also unshaded, so the shaded pinwheel has an area of $25 - 15 - 4 = 6$ square units.

Difficulty: Medium-hard

NCTM Standard: Geometry for Grades 6–8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Geometry > Plane Geometry > Squares > Square
Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Triangle

A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3 or 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

2007 AMC 8, Problem #24—
"A number is a multiple of three when the sum of its digits is a multiple of 3."

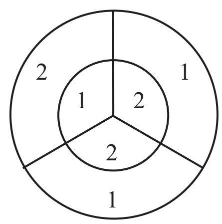
Solution (C) A number is a multiple of three when the sum of its digits is a multiple of 3. If the number has three distinct digits drawn from the set $\{1, 2, 3, 4\}$, then the sum of the digits will be a multiple of three when the digits are $\{1, 2, 3\}$ or $\{2, 3, 4\}$. That means the number formed is a multiple of three when, after the three draws, the number remaining in the bag is 1 or 4. The probability of this occurring is $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.

Difficulty: Hard

NCTM Standard: Data Analysis and Probability for Grades 6–8: understand and apply basic concepts of probability.

Mathworld.com Classification: Probability and Statistics > Probability > Probability

On the dart board shown in the figure, the outer circle has radius 6 and the inner circle has radius 3. Three radii divide each circle into three congruent regions, with point values shown. The probability that a dart will hit a given region is proportional to the area of the region. When two darts hit this board, the score is the sum of the point values in the regions. What is the probability that the score is odd?



(A) $\frac{17}{36}$ **(B)** $\frac{35}{72}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{37}{72}$ **(E)** $\frac{19}{36}$

2007 AMC 8, Problem #25—
"Find the area of each area."

Solution (B) The outer circle has area 36π and the inner circle has area 9π, making the area of the outer ring $36\pi - 9\pi = 27\pi$. So each region in the outer ring has area $\frac{27\pi}{3} = 9\pi$, and each region in the inner circle has area $\frac{9\pi}{3} = 3\pi$. The probability of hitting a given region in the inner circle is $\frac{3\pi}{36\pi} = \frac{1}{12}$, and the probability of hitting a given region in the outer ring is $\frac{9\pi}{36\pi} = \frac{1}{4}$. For the score to be odd, one of the numbers must be 1 and the other number must be 2. The probability of hitting a 1 is

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{12} = \frac{7}{12},$$

and the probability of hitting a 2 is

$$1 - \frac{7}{12} = \frac{5}{12}.$$

Therefore, the probability of hitting a 1 and a 2 in either order is

$$\frac{7}{12} \cdot \frac{5}{12} + \frac{5}{12} \cdot \frac{7}{12} = \frac{70}{144} = \frac{35}{72}.$$

Difficulty: Hard

NCTM Standard: Data Analysis and Probability for Grades 6–8: understand and apply basic concepts of probability.

Mathworld.com Classification: Geometry > Plane Geometry > Circles > Circle
Probability and Statistics > Probability > Probability