Connie multiplies a number by 2 and gets 60 as an answer. However, she should have divided the number by 2 to get the correct answer. What is the correct answer?

(A) 7.5 **(B)** 15 **(C)** 30 **(D)** 120 **(E)** 240

2005 AMC 8, Problem #1— "The original number times 2 is 60."

Solution (B) If a number multiplied by 2 results in 60, then the number must be 30. If 30 is divided by 2, the correct answer is 15.

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand meanings of operations and how they relate to one another

Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division > Division

Karl bought five three-ring binders from Pay-A-Lot at a cost of $\$2.50$ each. Pay-A-Lot had a 20%-off sale the following day. How much could Karl have saved on the purchase by waiting a day?

(A) $\$1.00$ **(B)** $\$2.00$ **(C)** $\$2.50$ **(D)** $\$2.75$ **(E)** $\$5.00$

2005 AMC 8, Problem #2—
"If he had purchased the binders a day later, he would have saved 20% of the total it cost him the first day, or $0.20\times$ **?"**

Solution (C) Karl spent $5 \times \$2.50 = \$12.50$ on the binders. If he had purchased the binders a day later, he would have saved 20% of this total, or $0.20 \times \$12.50 = \$2.50$.

OR

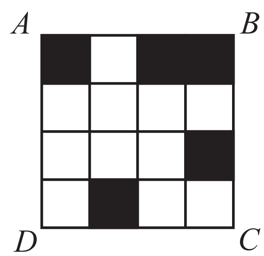
Karl could have bought five binders for the price of four in the the 20%-off sale, so he could have saved $\$2.50$.

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

What is the minimum number of small squares that must be colored black so the large square has diagonal BD as a line of symmetry?

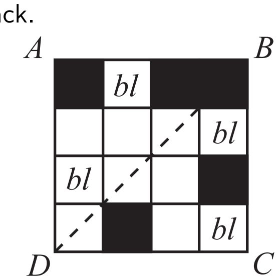


(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2005 AMC 8, Problem #3—
"Look at each of the existing 5 black squares. How many of them need another black square to be symmetric about BD?"

Solution

(D) For diagonal BD to be a line of symmetry in square $ABCD$, the four small squares labeled bl must be colored black.



Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6–8: apply transformations and use symmetry to analyze mathematical situations

Mathworld.com Classification: Geometry > Symmetry > Symmetry

A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.1 cm, 8.2 cm and 9.7 cm. What is the area of the square in square centimeters?

(A) 24 **(B)** 25 **(C)** 36 **(D)** 48 **(E)** 64

2005 AMC 8, Problem #4—
"What is the length of the side of the square?"

Solution
(C) The perimeter of the triangle is $6.1 + 8.2 + 9.7 = 24$ cm. The perimeter of the square is also 24 cm. Each side of the square is $24 \div 4 = 6$ cm. The area of the square is $6^2 = 36$ square centimeters.

Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Mathworld.com Classification: Calculus and Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter

Soda is sold in packs of 6, 12 and 24 cans. What is the minimum number of packs needed to buy exactly 90 cans of soda?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 15

2005 AMC 8, Problem #5—
"First buy as many 24-packs as possible, then 12, and lastly 6."

Solution
(B) To get the minimum total number, purchase as many 24-packs as possible: three 24-packs contain 72 cans, and $90 - 72 = 18$. To get the remaining 18 cans, purchase one 12-pack and one 6-pack. The minimum total number of packs is 5.

Difficulty: Medium-Easy
NCTM Standard: Problem Solving Standard for Grades 6–8: solve problems that arise in mathematics and in other contexts
Mathworld.com Classification: Number Theory > Congruences > Mod

Suppose d is a digit. For how many values of d is $2.00d5 > 2.005$?

(A) 0 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 10

2005 AMC 8, Problem #6— "What is the smallest d such that $2.00d5 > 2.005$?"

Solution
(C) The number $2.00d5$ is greater than 2.005 for $d = 5$, 6, 7, 8 and 9. Therefore, there are five digits satisfying the inequality.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems

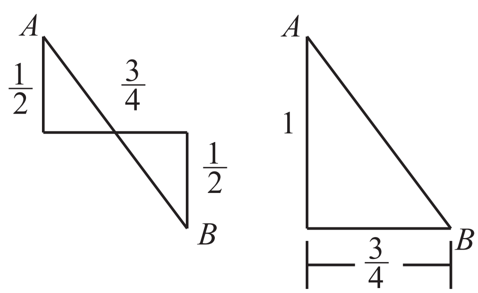
Mathworld.com Classification: Number Theory > Arithmetic > Number Bases > Digit

Bill walks $\frac{1}{2}$ mile south, then $\frac{3}{4}$ mile east, and finally $\frac{1}{2}$ mile south. How many miles is he, in a direct line, from his starting point?

(A) 1 **(B)** $1\frac{1}{4}$ **(C)** $1\frac{1}{2}$ **(D)** $1\frac{3}{4}$ **(E)** 2

2005 AMC 8, Problem #7— "Find the hypotenuse of a triangle with legs $\frac{1}{2}+\frac{1}{2}$ and $\frac{3}{4}$."

Solution
(B) The diagram on the left shows the path of Bill's walk. As the diagram on the right illustrates, he could also have walked from A to B by first walking 1 mile south then $\frac{3}{4}$ mile east.



By the Pythagorean Theorem,

$$(AB)^2 = 1^2 + \left(\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{25}{16},$$

so $AB = \frac{5}{4} = 1\frac{1}{4}$.

Difficulty: Medium-hard
NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
Mathworld.com Classification: Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Right Triangle

Suppose m and n are odd positive integers. Which of the following must also be an odd integer?

(A) $m+3n$ **(B)** $3m-n$ **(C)** $3mn$ **(D)** $(nm+3)^2$ **(E)** $3m^2+3n^2$

2005 AMC 8, Problem #8— "For an example, let $m = n = 1$."

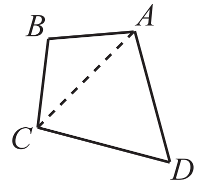
Solution
(C) If $m = n = 1$, then

$$m + 3n = 4, \quad 3m - n = 2, \quad 3mn = 3, \quad (mn + 3)^2 = 16 \text{ and } 3m^2 + 2n^2 = 6.$$

This shows that (A), (B), (D) and (E) can be even when m and n are odd. On the other hand, because the product of odd integers is always odd, $3mn$ is always odd if m and n are odd.
Question: Which of the expressions are always even if m and n are odd? What are the possibilities if m and n are both even? If one is even and the other odd?

Difficulty: Medium
NCTM Standard: Algebra Standard for Grades 6–8: understand patterns, relations, and functions
Mathworld.com Classification: Number Theory > Parity > Odd Number

In quadrilateral $ABCD$ sides $\overline{AB}$ and $\overline{BC}$ both have length 10, sides $\overline{CD}$ and $\overline{DA}$ both have length 17, and the measure of angle ADC is 60°. What is the length of diagonal $\overline{AC}$?



(A) 13.5 **(B)** 14 **(C)** 15.5 **(D)** 17 **(E)** 18.5

2005 AMC 8, Problem #9—
"What can be said about triangle ACD?"

Solution
(D) Triangle ACD is an isosceles triangle with a 60° angle, so it is also equilateral. Therefore, the length of $\overline{AC}$ is 17.

Difficulty: Medium
NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
Mathworld.com Classification: Geometry > Plane Geometry > Quadrilaterals > Quadrilateral

Joe had walked half way from home to school when he realized he was late. He ran the rest of the way to school. He ran 3 times as fast as he walked. Joe took 6 minutes to walk half way to school. How many minutes did it take Joe to get from home to school?

(A) 7 **(B)** 7.3 **(C)** 7.7 **(D)** 8 **(E)** 8.3

2005 AMC 8, Problem #10— "Covering the same distance three times as fast takes one-third the time."

Solution

(D) Covering the same distance three times as fast takes one-third the time. So Joe ran for 2 minutes. His total time was $6 + 2 = 8$ minutes.

Difficulty: Easy
NCTM Standard: Problem Solving Standard for Grades 6–8: solve problems that arise in mathematics and in other contexts
Mathworld.com Classification:
Calculus and Analysis > Differential Geometry > Differential Geometry of Curves > Velocity

The sales tax rate in Bergville is 6%. During a sale at the Bergville Coat Closet, a coat is discounted 20% off its $\$90.00$ price. Two clerks, Jack and Jill, calculate the bill independently. Jack rings up $\$90.00$ and adds 6% sales tax, then subtracts 20% from this total. Jill rings up $\$90.00$, subtracts 20% of the price, then adds 6% of the discounted price for sales tax. What is Jack's total minus Jill's total?

(A) $-\$1.06$ **(B)** $-\$0.53$ **(C)** 0 **(D)** $\$0.53$ **(E)** $\$1.06$

2005 AMC 8, Problem #11—
"To add 6% sales tax to an item, multiply the price by 1.06."

Solution
(C) To add 6% sales tax to an item, multiply the price by 1.06. To calculate a 20% discount, multiply the price by 0.8. Because both actions require only mutliplication, and because multiplication is commutative, the order of operations doesn't matter. Jack and Jill will get the same total.
Note: Jack's final computation is $0.80(1.06 \times \$90.00)$ and Jill's is $1.06(0.80 \times \$90.00)$. Both yield the same product, $\$76.32$.

Difficulty: Medium-easy
NCTM Standard: Number and Operations Standard for Grades 6–8: understand meanings of operations and how they relate to one another
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

Big Al, the ape, ate 100 bananas from May 1 through May 5. Each day he ate six more bananas than on the previous day. How many bananas did Big Al eat on May 5?

(A) 20 **(B)** 22 **(C)** 30 **(D)** 32 **(E)** 34

2005 AMC 8, Problem #12—
"If Big Al had eaten 10 bananas on May 1, how many would he have eaten on May 2?"

Solution

(D) If Big Al had eaten 10 bananas on May 1, then he would have eaten $10 + 16 + 22 + 28 + 34 = 110$ bananas. This is 10 bananas too many, so he actually ate 2 fewer bananas each day. Thus, Big Al ate 8 bananas on May 1 and 32 bananas on May 5.

OR

The average number of bananas eaten per day was $\frac{100}{5} = 20$. On May 4, Big Al ate six more bananas than on May 3, and on May 2 he ate six fewer bananas than on May 3. Similarly, on May 5 Big Al ate twelve more bananas than on May 3, and on May 1 he ate twelve fewer bananas. Therefore, the average number of bananas he ate per day, 20, is equal to the number of bananas he ate on May 3. So on May 5 Big Al ate $20 + 12 = 32$ bananas.

OR

Let x be the number of bananas that Big Al ate on May 5. The following chart documents his banana intake for the five days.

May 5	May 4	May 3	May 2	May 1
x	$x - 6$	$x - 12$	$x - 18$	$x - 24$

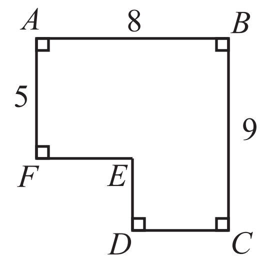
The total number of bananas Big Al ate was $5x - 60$, which must be 100. So Big Al ate $x = \frac{160}{5} = 32$ bananas on May 5.

Difficulty: Medium

NCTM Standard: Algebra Standard for Grades 6–8: understand patterns, relations, and functions

Mathworld.com Classification: Number Theory > Sequences > Sequence

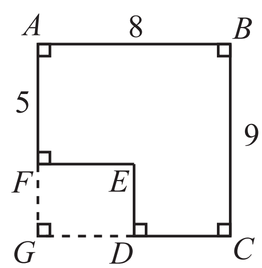
The area of polygon $ABCDEF$ is 52 with $AB = 8$, $BC = 9$ and $FA = 5$. What is $ED + EF$?



(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

2005 AMC 8, Problem #13— "What is the area of rectangle $ABCG$ and rectangle $FEDG$?"

Solution
(C)



Rectangle $ABCG$ has area $8 \times 9 = 72$, so rectangle $FEDG$ has area $72 - 52 = 20$. The length of $\overline{FG}$ is $9 - 5 = 4$, so the length of $\overline{EF}$ is $\frac{20}{4} = 5$. Therefore, $ED + EF = 4 + 5 = 9$.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: use visualization, spatial reasoning, and geometric modeling to solve problems

Mathworld.com Classification: Geometry > Plane Geometry > Polygons > Polygon

The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

(A) 80 **(B)** 96 **(C)** 100 **(D)** 108 **(E)** 192

2005 AMC 8, Problem #14—
"How many total games does each team play?"

Solution
(B) Each team plays 10 games in its own division and 6 games against teams in the other division. So each of the 12 teams plays 16 conference games. Because each game involves two teams, there are $\frac{12\times16}{2} = 96$ games scheduled.

Difficulty: Medium-hard
NCTM Standard: Problem Solving Standard for Grades 6–8: use solve problems that arise in mathematics and in other contexts
Mathworld.com Classification: Discrete Mathematics > Combinatorics > Permutations > Combination

How many different isosceles triangles have integer side lengths and perimeter 23?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 9 **(E)** 11

2005 AMC 8, Problem #15— "The length of the base is less than the sum of the two sides of equal length."

Solution

(C) Because the perimeter of such a triangle is 23, and the sum of the two equal side lengths is even, the length of the base is odd. Also, the length of the base is less than the sum of the other two side lengths, so it is less than half of 23. Thus the six possible triangles have side lengths 1, 11, 11; 3, 10, 10; 5, 9, 9; 7, 8, 8; 9, 7, 7 and 11, 6, 6.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: use analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Mathworld.com Classification: Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Isosceles Triangle

A five-legged Martian has a drawer full of socks, each of which is red, white or blue, and there are at least five socks of each color. The Martian pulls out one sock at a time without looking. How many socks must the Martian remove from the drawer to be certain there will be 5 socks of the same color?

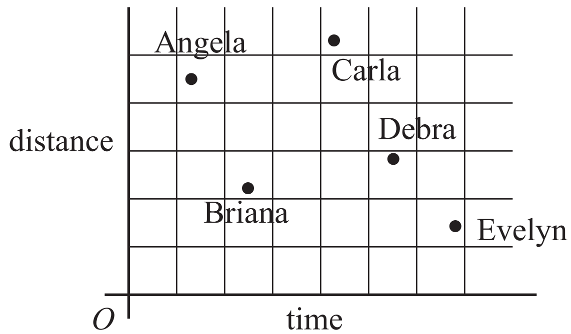
(A) 6 **(B)** 9 **(C)** 12 **(D)** 13 **(E)** 15

2005 AMC 8, Problem #16—
"At most, how many socks of each color can the Martian pull out without having a set?"

Solution
(D) It is possible for the Martian to pull out at most 4 red, 4 white and 4 blue socks without having a matched set. The next sock it pulls out must be red, white or blue, which gives a matched set. So the Martian must select $4 \times 3 + 1 = 13$ socks to be guaranteed a matched set of five socks.

Difficulty: Hard
NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: use understand and apply basic concepts of probability
Mathworld.com Classification: Probability and Statistics > Probability > Probability

The results of a cross-country team's training run are graphed below. Which student has the greatest average speed?



(A) Angela **(B)** Briana **(C)** Carla **(D)** Debra **(E)** Evelyn

2005 AMC 8, Problem #17—
"The ratio of distance to time, or average speed, is indicated by the slope of the line from the origin to each runner's point in the graph."

Solution
(A) Angela covered more distance in less time than Briana, Debra and Evelyn. So her average speed is greater than any of their average speeds. Angela went almost as far as Carla in less than half the time that it took Carla. So Angela's average speed is also greater than Carla's.

OR

The ratio of distance to time, or average speed, is indicated by the slope of the line from the origin to each runner's point in the graph. Therefore, the line from the origin with the greatest slope will correspond to the runner with the greatest average speed. Because Angela's line has the greatest slope, she has the greatest average speed.

Difficulty: Medium

NCTM Standard: Measurement Standard for Grades 6–8: use understand measurable attributes of objects and the units, systems, and processes of measurement

Mathworld.com Classification: Algebra > Vector Algebra > Speed

How many three-digit numbers are divisible by 13?

(A) 7 **(B)** 67 **(C)** 69 **(D)** 76 **(E)** 77

2005 AMC 8, Problem #18— "What are the smallest and largest three-digit numbers divisible by 13?"

Solution

(C) The smallest three-digit number divisible by 13 is $13 \times 8 = 104$, so there are seven two-digit multiples of 13. The greatest three-digit number divisible by 13 is $13 \times 76 = 988$. Therefore, there are $76 - 7 = 69$ three-digit numbers divisible by 13.

OR

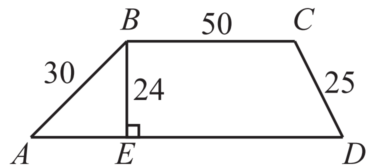
Because the integer part of $\frac{999}{13}$ is 76, there are 76 multiples of 13 less than or equal to 999. Because the integer part of $\frac{99}{13}$ is 7, there are 7 multiples of 13 less than or equal to 99. That means there are $76 - 7 = 69$ multiples of 13 between 100 and 999.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6–8: understand numbers, ways of representing numbers, relationships among numbers, and number systems

Mathworld.com Classification: Number Theory > Divisors > Divisible

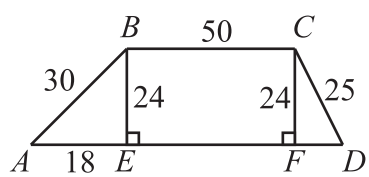
What is the perimeter of trapezoid $ABCD$?



(A) 180 **(B)** 188 **(C)** 196 **(D)** 200 **(E)** 204

2005 AMC 8, Problem #19—
"Use the Pythagorean Theorem to solve for AE."

Solution
(A)



By the Pythagorean Theorem, $AE = \sqrt{30^2 - 24^2} = \sqrt{324} = 18$. (Or note that triangle AEB is similar to a 3-4-5 right triangle, so $AE = 3 \times 6 = 18$.)
It follows that $CF = 24$ and $FD = \sqrt{25^2 - 24^2} = \sqrt{49} = 7$. The perimeter of the trapezoid is $50 + 30 + 18 + 50 + 7 + 25 = 180$.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Mathworld.com Classification: Geometry > Plane Geometry > Quadrilaterals > Trapezoid

Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

(A) 6 **(B)** 8 **(C)** 12 **(D)** 14 **(E)** 24

2005 AMC 8, Problem #20—
"Write the points where Alice and Bob will stop after each move and compare points."

Solution
(A) Write the points where Alice and Bob will stop after each move and compare points.

Move	0	1	2	3	4	5	6
Alice:	12	5	10	3	8	1	6
Bob:	12	3	6	9	12	3	6

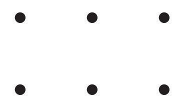
So Alice and Bob will be together again after six moves.

OR

If Bob does not move and Alice moves $9 + 5 = 14$ points or 2 points each time, they will still be in the same relative position from each other after each turn. If Bob does not move, they will be on the same point when Alice first stops on point 12, where she started. So Alice will have to move 2 steps 6 times to stop at her starting point.

Difficulty: Medium-hard
NCTM Standard: Problem Solving Standard for Grades 6–8: solve problems that arise in mathematics and in other contexts
Mathworld.com Classification: Number Theory > Congruences > Residue

How many distinct triangles can be drawn using three of the dots below as vertices?



(A) 9 **(B)** 12 **(C)** 18 **(D)** 20 **(E)** 24

2005 AMC 8, Problem #21—
"To make a triangle, select as vertices two dots from one row and one from the other row."

Solution
(C) To make a triangle, select as vertices two dots from one row and one from the other row. To select two dots in the top row, decide which dot is not used. This can be done in three ways. There are also three ways to choose one dot to use from the bottom row. So there are $3 \times 3 = 9$ triangles with two vertices in the top row and one in the bottom. Similarly, there are nine triangles with one vertex in the top row and two in the bottom. This gives a total of $9 + 9 = 18$ triangles.

Difficulty: Medium-hard

NCTM Standard: Problem Solving Standard for Grades 6–8: apply and adapt a variety of appropriate strategies to solve problems

Mathworld.com Classification: Discrete Mathematics > Combinatorics > Permutations > Permutation

A company sells detergent in three different sized boxes: small (S), medium (M) and large (L). The medium size costs 50% more than the small size and contains 20% less detergent than the large size. The large size contains twice as much detergent as the small size and costs 30% more than the medium size. Rank the three size from best to worst buy.

(A) SML **(B)** MLS **(C)** MSL **(D)** LSM **(E)** LMS

2005 AMC 8, Problem #22—
"For convenience, suppose the small size costs $1 and weighs 10 ounces. How much per ounce is the small size? Solve for medium and large."

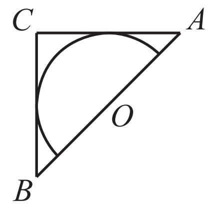
Solution
(B) Neither the units of size nor the cost are important in this problem. So for convenience, suppose the small size costs $1 and weighs 10 ounces. To determine the relative value, we compare the cost per unit weight.

$$S : \frac{\$1.00}{10} = 10¢ \text{ per oz.} \quad M : \frac{\$1.50}{0.8 \times 20} = 9.375¢ \text{ per oz.} \quad L : \frac{1.3 \times \$1.50}{20} = 9.75¢ \text{ per oz.}$$

So the value, or buy, from best to worst is medium, large and small.

Difficulty: Hard
NCTM Standard: Data Analysis and Probability Standard for Grades 6–8: develop and evaluate inferences and predictions that are based on data
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

Isosceles right triangle ABC encloses a semicircle of area 2π. The circle has its center O on hypotenuse $\overline{AB}$ and is tangent to sides $\overline{AC}$ and $\overline{BC}$. What is the area of triangle ABC?



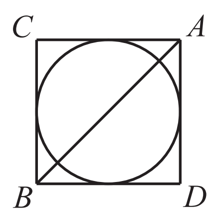
(A) 6 **(B)** 8 **(C)** 3π **(D)** 10 **(E)** 4π

2005 AMC 8, Problem #23—

"Reflect the triangle and the semicircle across the hypotenuse $\overline{AB}$ to obtain a circle inscribed in a square. What is the radius?"

Solution

(B) Reflect the triangle and the semicircle across the hypotenuse $\overline{AB}$ to obtain a circle inscribed in a square. The circle has area 4π. The radius of a circle with area 4π is 2. The side length of the square is 4 and the area of the square is 16. So the area of the triangle is 8.



Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Mathworld.com Classification: Geometry > Line Geometry > Concurrence > Inscribed

A certain calculator has only two keys $[+1]$ and $[\times 2]$. When you press one of the keys, the calculator automatically displays the result. For instance, if the calculator originally displayed "9" and you pressed $[+1]$, it would display "10." If you then pressed $[\times 2]$, it would display "20." Starting with the display "1," what is the fewest number of keystrokes you would need to reach "200"?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

2005 AMC 8, Problem #24—
"One way to solve the problem is to work backward, either dividing by 2 if the number is even or subtracting 1 if the number is odd."

Solution
(B) One way to solve the problem is to work backward, either dividing by 2 if the number is even or subtracting 1 if the number is odd.

$$200/2 \to 100/2 \to 50/2 \to 25-1 \to 24/2 \to 12/2 \to 6/2 \to 3-1 \to 2/2 \to 1$$

So if you press $[\times 2]$ $[+1]$ $[\times 2]$ $[\times 2]$ $[\times 2]$ $[+1]$ $[\times 2]$ $[\times 2]$ $[\times 2]$ or 9 keystrokes, you can reach 200 from 1.
To see that no sequence of eight keystrokes works, begin by noting that of the four possible sequences of two keystrokes, $[\times 2]$ $[\times 2]$ produces the maximum result. Furthermore, $[+1]$ $[\times 2]$ produces a result larger than either $[\times 2]$ $[+1]$ or $[+1]$ $[+1]$. So the largest possible result of a sequence of eight keystrokes is "256," produced by either

$$[\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]$$

or

$$[+1]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2].$$

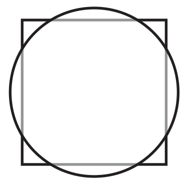
The second largest results is "192," produced by

$$[\times 2]\ [+1]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2]\ [\times 2].$$

Thus no sequence of eight keystrokes produces a result of "200."

Difficulty: Hard
NCTM Standard: Problem Solving Standard for Grades 6–8: solve problems that arise in mathematics and in other contexts
Mathworld.com Classification: Applied Mathematics > Optimization > Global Optimization

A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the area of the regions that are outside the circle and inside the square. What is the radius of the circle?



(A) $\frac{2}{\sqrt{\pi}}$ **(B)** $\frac{1+\sqrt{2}}{2}$ **(C)** $\frac{3}{2}$ **(D)** $\sqrt{3}$ **(E)** $\sqrt{\pi}$

2005 AMC 8, Problem #25—

"Because the circle and square share the same interior region and the area of the two exterior regions indicated are equal, the square and the circle must have equal area."

Solution

(A) Because the circle and square share the same interior region and the area of the two exterior regions indicated are equal, the square and the circle must have equal area. The area of the square is 2^2 or 4. Because the area of both the circle and the square is 4, $4 = \pi r^2$. Solving for r yields $r^2 = \frac{4}{\pi}$, so $r = \sqrt{\frac{4}{\pi}} = \frac{2}{\sqrt{\pi}}$.

Note: It is not necessary that the circle and square have the same center.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6–8: use visualization, spatial reasoning, and geometric modeling to solve problems

Mathworld.com Classification: Geometry > Plane Geometry > Circles > Circle