- On a map, a 12-centimeter length represents 72 kilometers. How many kilometers does a 17-centimeter length represent?

(A) 6 **(B)** 102 **(C)** 204 **(D)** 864 **(E)** 1224

2004 AMC 8, Problem #1
"How many kilometers does 1 centimeter represent?"

- **Solution**

  (B) If 12 centimeters represents 72 kilometers, then 1 centimeter represents 6 kilometers. So 17 centimeters represents $17 \times 6 = 102$ kilometers.

Difficulty: Easy
NCTM Standard: Measurement:
apply appropriate techniques, tools, and formulas to determine measurements
Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Directly Proportional

- How many different four-digit numbers can be formed by rearranging the four digits in 2004?

(A) 4 **(B)** 6 **(C)** 16 **(D)** 24 **(E)** 81

2004 AMC 8, Problem #2
"Which digits can be in the thousands place?"

- **Solution (B)** To form a four-digit number using 2, 0, 0 and 4, the digit in the thousands place must be 2 or 4. There are three places available for the remaining nonzero digit, whether it is 4 or 2. So the final answer is 6.

OR

Make a list: 2004, 2040, 2400, 4002, 4020 and 4200. So 6 numbers are possible.

Difficulty: Medium

NCTM Standard: understand numbers, ways of representing numbers, relationships among numbers, and number systems

Mathworld.com Classification:
Number Theory > Arithmetic > Number Bases > Digit

- Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they share, how many meals should they have ordered to have just enough food for the 12 of them?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 15 **(E)** 18

2004 AMC 8, Problem #3
"Find the ratio of food to people."

- **Solution (A)** If 12 people order $\frac{18}{12} = 1\frac{1}{2}$ times too much food, they should have ordered $\frac{12}{\frac{3}{2}} = \frac{2}{3} \times 12 = 8$ meals.

OR

Let x be the number of meals they should have ordered. Then,

$$\frac{12}{18} = \frac{x}{12},$$

so

$$x = 8.$$

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6–8: Understand and use ratios and proportions to represent quantitative relationships.

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Ratio

- Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament.

  Lance, Sally, Joy and Fred are chosen for the team. In how many ways can the three starters be chosen?

  (A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

2004 AMC 8, Problem #4
"When three players start, one is the alternate."

- **Solution**

  (B) When three players start, one is the alternate. Because any of the four players might be the alternate, there are four ways to select a starting team: Lance-Sally-Joy, Lance-Sally-Fred, Lance-Joy-Fred and Sally-Joy-Fred.

Difficulty: Medium

NCTM Standard: Number and Operations
understand numbers, ways of representing numbers, relationships among numbers, and number systems

Mathworld.com Classification:
Discrete Mathematics > Combinatorics > Permutations > Combination

- Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament.

  The losing team of each game is eliminated from the tournament. If sixteen teams compete, how many games will be played to determine the winner?

  (A) 4 **(B)** 7 **(C)** 8 **(D)** 15 **(E)** 16

2004 AMC 8, Problem #5
"How many teams need to lose in order for one team to be left?

- **Solution**
  (D) It takes 15 games to eliminate 15 teams.

Difficulty: Medium
NCTM Standard: Data Analysis and Probability
develop and evaluate inferences and predictions that are based on data
Mathworld.com Classification:
Applied Mathematics > Game Theory > Game

- Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament.

  After Sally takes 20 shots, she has made 55% of her shots. After she takes 5 more shots, she raises her percentage to 56%. How many of the last 5 shots did she make?

  (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2004 AMC 8, Problem #6
"How many shots did she make at first?"

- **Solution**

  (C) If Sally makes 55% of her 20 shots, she makes $0.55 \times 20 = 11$ shots. If Sally makes 56% of her 25 shots, she makes $0.56 \times 25 = 14$ shots. So she makes $14 - 11 = 3$ of the last 5 shots.

Difficulty: Medium

NCTM Standard: Number and Operations
understand meanings of operations and how they relate to one another

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Percent

- An athlete's target heart rate, in beats per minute, is 80% of the theoretical maximum heart rate. The maximum heart rate is found by subtracting the athlete's age, in years, from 220. To the nearest whole number, what is the target heart rate of an athlete who is 26 years old?

(A) 134 **(B)** 155 **(C)** 176 **(D)** 194 **(E)** 243

2004 AMC 8, Problem #7

"Take 80% of $220 - 26$."

- **Solution**

(B) A 26-year-old's target heart rate is $0.8(220 - 26) = 155.2$ beats per minute. The nearest whole number is 155.

Difficulty: Medium

NCTM Standard: Number and Operations
understand meanings of operations and how they relate to one another

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Percent

- Find the number of two-digit positive integers whose digits total 7.

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

2004 AMC 8, Problem #8
"List the two-digit numbers whose digits sum to 7."

- **Solution**

  (B) There are 7 two-digit numbers whose digits sum to 7: 16, 61, 25, 52, 34, 43 and 70.

Difficulty: Medium

NCTM Standard: Number and Operations
understand numbers, ways of representing numbers, relationships among numbers, and number systems

Mathworld.com Classification:
Number Theory > Arithmetic > Number Bases > Digit

- The average of the five numbers in a list is 54. The average of the first two numbers is 48. What is the average of the last three numbers?

(A) 55 **(B)** 56 **(C)** 57 **(D)** 58 **(E)** 59

2004 AMC 8, Problem #9

"Find the sum of all five numbers and the sum of the first two numbers."

- **Solution (D)** The sum of all five numbers is $5 \times 54 = 270$. The sum of the first two numbers is $2 \times 48 = 96$, so the sum of the last three numbers is $270 - 96 = 174$. The average of the last three numbers is $\frac{174}{3} = 58$.

Difficulty: Medium-hard

NCTM Standard: Data Analysis and Probability

select and use appropriate statistical methods to analyze data

Mathworld.com Classification:

Calculus and Analysis > Special Functions > Means > Arithmetic Mean

- Handy Aaron helped a neighbor $1\frac{1}{4}$ hours on Monday, 50 minutes on Tuesday, from 8:20 to 10:45 on Wednesday morning, and a half-hour on Friday. He is paid \$3 per hour. How much did he earn for the week?

(A) \$8 **(B)** \$9 **(C)** \$10 **(D)** \$12 **(E)** \$15

2004 AMC 8, Problem #10
"Convert the time he spent all into minutes and then into hours."

- **Solution (E)** Aaron worked 75 minutes on Monday, 50 on Tuesday, 145 on Wednesday and 30 on Friday, for a total of 300 minutes or 5 hours. He earned $5 \times \$3 = \$15$.

Difficulty: Medium
NCTM Standard: Data Analysis and Probability
develop and evaluate inferences and predictions that are based on data
Mathworld.com Classification:
Number Theory > Arithmetic > Number Bases > Sexagesimal

- The numbers -2, 4, 6, 9 and 12 are rearranged according to these rules:

  1. The largest isn't first, but it is in one of the first three places.
  2. The smallest isn't last, but it is in one of the last three places.
  3. The median isn't first or last.

  What is the average of the first and last numbers?

  (A) 3.5 **(B)** 5 **(C)** 6.5 **(D)** 7.5 **(E)** 8

2004 AMC 8, Problem #11
"What numbers have the possibility of being in the first and last places?"

- **Solution**

  (C) The largest, smallest and median occupy the three middle places, so the other two numbers, 9 and 4, are in the first and last places. The average of 9 and 4 is $\frac{9+4}{2} = 6.5$.

Difficulty: Medium
NCTM Standard: Data Analysis and Probability
select and use appropriate statistical methods to analyze data
Mathworld.com Classification:
Probability and Statistics > Rank Statistics > Median

- Niki usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on 9 hours, and during that time she has used it for 60 minutes. If she doesn't talk any more but leaves the phone on, how many more hours will the battery last?

(A) 7 **(B)** 8 **(C)** 11 **(D)** 14 **(E)** 15

2004 AMC 8, Problem #12
"The phone has been used for 1 hour to talk, how much of the battery has it used?"

- **Solution**

(B) The phone has been used for 60 minutes, or 1 hour, to talk, during which time it has used $\frac{1}{3}$ of the battery. In addition, the phone has been on for 8 hours without talking, which used an additional $\frac{8}{24}$ or $\frac{1}{3}$ of the battery. Consequently, $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ of the battery has been used, meaning that $\frac{1}{3}$ of the battery, or $\frac{1}{3} \times 24 = 8$ hours remain if Niki does not talk on her phone.

OR

Niki's battery has 24 hours of potential battery life. By talking for one hour, she uses $\frac{1}{3} \times 24 = 8$ hours of battery life. In addition, the phone is left on and unused for 8 hours, using an additional 8 hours. This leaves $24 - 8 - 8 = 8$ hours of battery life if the phone is on and unused.

Difficulty: Medium-hard

NCTM Standard: Measurement Standard for Grades 6–8: use mathematical models to represent and understand quantitative relationships

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Ratios

- Amy, Bill and Celine are friends with different ages. Exactly one of the following statements is true.

  I. Bill is the oldest.

  II. Amy is not the oldest.

  III. Celine is not the youngest.

  Rank the friends from the oldest to the youngest.

  (A) Bill, Amy, Celine **(B)** Amy, Bill, Celine **(C)** Celine, Amy, Bill
  (D) Celine, Bill, Amy **(E)** Amy, Celine, Bill

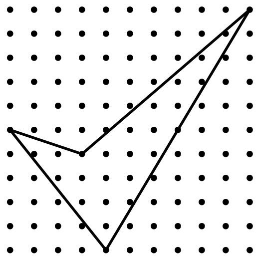
2004 AMC 8, Problem #13
"Try assuming I is true and II and III are false. Do the same for II and III."

- **Solution**

  (E) Bill is not the oldest, because if he were, the first two statements would be true. Celine is not the oldest, because if she were, the last two statements would be true. Therefore, Amy is the oldest. So the first two statements are false. The last statement must be true. This means that Celine is not the youngest, so Bill is the youngest. The correct order from oldest to youngest is Amy, Celine, Bill.

Difficulty: Medium-hard
NCTM Standard: Problem Solving
monitor and reflect on the process of mathematical problem solving
Mathworld.com Classification:
Foundations of Mathematics > Logic > General Logic > True

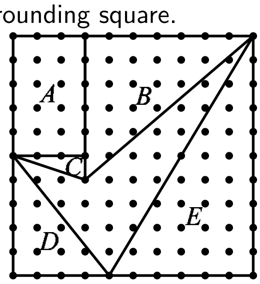
- What is the area enclosed by the geoboard quadrilateral below?



(A) 15 **(B)** $18\frac{1}{2}$ **(C)** $22\frac{1}{2}$ **(D)** 27 **(E)** 41

2004 AMC 8, Problem #14
"Use the area of the surrounding square."

- **Solution (C)** To find the area, subtract the areas of regions A, B, C, D and E from that of the surrounding square.

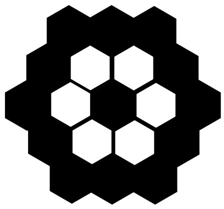


Square: $10 \times 10 = 100$
Region A: $3 \times 5 = 15$
Region B: $\frac{1}{2} \times 6 \times 7 = 21$
Region C: $\frac{1}{2} \times 1 \times 3 = 1\frac{1}{2}$
Region D: $\frac{1}{2} \times 4 \times 5 = 10$
Region E: $\frac{1}{2} \times 6 \times 10 = 30$

The area is $100 - \left(15 + 21 + 1\frac{1}{2} + 10 + 30\right) = 100 - 77\frac{1}{2} = 22\frac{1}{2}$ square units.

Difficulty: Medium-hard
NCTM Standard: Geometry
specify locations and describe spatial relationships using coordinate geometry and other representational systems
Mathworld.com Classification:
Discrete Mathematics > Point Lattices > Geoboard

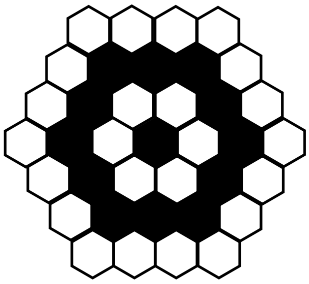
- Thirteen black and six white hexagonal tiles were used to create the figure below. If a new figure is created by attaching a border of white tiles with the same size and shape as the others, what will be the difference between the total number of white tiles and the total number of black tiles in the new figure?



(A) 5 **(B)** 7 **(C)** 11 **(D)** 12 **(E)** 18

2004 AMC 8, Problem #15
"Count the tiles in the attached border."

- **Solution (C)** The next border requires an additional $6 \times 3 = 18$ white tiles. A total of 24 white and 13 black tiles will be used, so the difference is $24 - 13 = 11$.



Difficulty: Medium-hard
NCTM Standard: Geometry
use visualization, spatial reasoning, and geometric modeling to solve problems
Mathworld.com Classification:
Geometry > Plane Geometry > Tiling > Tiling

- Two 600 ml pitchers contain orange juice. One pitcher is $\frac{1}{3}$ full and the other pitcher is $\frac{2}{5}$ full. Water is added to fill each pitcher completely, then both pitchers are poured into one large container. What fraction of the mixture in the large container is orange juice?

(A) $\frac{1}{8}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{11}{30}$ **(D)** $\frac{11}{19}$ **(E)** $\frac{11}{15}$

2004 AMC 8, Problem #16
"Convert the amount of orange juice in each pitcher into milliliters."

- **Solution (C)** Because the first pitcher was $\frac{1}{3}$ full of orange juice, after filling with water it contains 200 ml of juice and 400 ml of water. Because the second pitcher was $\frac{2}{5}$ full of orange juice, after filling it contains 240 ml of orange juice and 360 ml of water. In all, the amount of orange juice is 440 ml out of a total of 1200 ml or $\frac{440}{1200} = \frac{11}{30}$ of the mixture.

Difficulty: Hard

NCTM Standard: Measurement
understand measurable attributes of objects and the units, systems, and processes of measurement

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions

- Three friends have a total of 6 identical pencils, and each one has at least one pencil. In how many ways can this happen?

(A) 1 **(B)** 3 **(C)** 6 **(D)** 10 **(E)** 12

2004 AMC 8, Problem #17
"How many ways can one friend have 4 pencils (or 3, or 2)?"

- **Solution**

(D) The largest number of pencils that any friend can have is four. There are 3 ways that this can happen: $(4, 1, 1)$, $(1, 4, 1)$ and $(1, 1, 4)$. There are 6 ways one person can have 3 pencils: $(3, 2, 1)$, $(3, 1, 2)$, $(2, 3, 1)$, $(2, 1, 3)$, $(1, 2, 3)$ and $(1, 3, 2)$. There is only one way all three can have two pencils each: $(2, 2, 2)$. The total number of possibilities is $3 + 6 + 1 = 10$.

OR

The possible distributions of 6 pencils among 3 friends are the following:

1	1	4
1	2	3
1	3	2
1	4	1
2	1	3
2	2	2
2	3	1
3	1	2
3	2	1
4	1	1

The number of possible distributions is 10.

Difficulty: Medium-hard

NCTM Standard: Problem Solving
solve problems that arise in mathematics and in other contexts

Mathworld.com Classification:
Discrete Mathematics > Combinatorics > Partitions > Partition

- Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value. The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points. Who hits the region worth 6 points?

(A) Alice **(B)** Ben **(C)** Cindy **(D)** Dave **(E)** Ellen

2004 AMC 8, Problem #18
"Start with Ben, what could Ben have hit? Then solve for Cindy, etc.

- **Solution**

(A) Ben must hit 1 and 3. This means Cindy must hit 5 and 2, because she scores 7 using two different numbers, neither of which is 1 or 3. By similar reasoning, Alice hits 10 and 6, Dave hits 7 and 4, and Ellen hits 9 and 8. Alice hits the 6.

OR

Ellen's score can be obtained by either $10 + 7$ or $9 + 8$. In the first case, it is impossible for Alice to score 16. So Ellen's 17 is obtained by scoring 9 and 8, and Alice's total of 16 is the result of her hitting 10 and 6. The others scored $11 = 7 + 4$, $7 = 5 + 2$ and $4 = 3 + 1$.

Difficulty: Medium
NCTM Standard: Problem Solving
apply and adapt a variety of appropriate strategies to solve problems
Mathworld.com Classification:
Discrete Mathematics > Combinatorics > Partitions > Partition

- A whole number larger than 2 leaves a remainder of 2 when divided by each of the numbers 3, 4, 5 and 6. The smallest such number lies between which two numbers?

(A) 40 and 49 **(B)** 60 and 79 **(C)** 100 and 129 **(D)** 210 and 249 **(E)** 320 and 369

2004 AMC 8, Problem #19
"What is the smallest whole number evenly divisible by 3, 4, 5, and 6?"

- **Solution (B)** The numbers that leave a remainder of 2 when divided by 4 and 5 are 22, 42, 62 and so on. Checking these numbers for a remainder of 2 when divided by both 3 and 6 yields 62 as the smallest.

OR

The smallest whole number that is evenly divided by each of 3, 4, 5 and 6 is LCM$\{3, 4, 5, 6\} = 2^2 \times 3 \times 5 = 60$. So the smallest whole number greater than 2 that leaves a remainder of 2 when divided by each of 3, 4, 5 and 6 is 62.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6–8: Use factors, multiples, prime factorization, and relatively prime numbers to solve problems.

Mathworld.com Classification:
Number Theory > Arithmetic > Multiplication and Division > Remainder

- Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 27 **(E)** 36

2004 AMC 8, Problem #20

"The 6 empty chairs are $\frac{1}{4}$ of the chairs in the room."

- **Solution (D)** Because the 6 empty chairs are $\frac{1}{4}$ of the chairs in the room, there are $6 \times 4 = 24$ chairs in all. The number of seated people is $\left(\frac{3}{4}\right)24 = 18$, and this is $\frac{2}{3}$ of the people present. It follows that

$$\frac{18}{\text{people present}} = \frac{2}{3}.$$

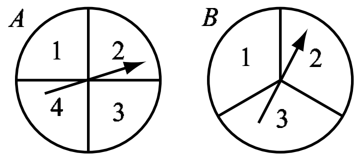
So there are 27 people in the room.

Difficulty: Medium-hard

NCTM Standard: Problem Solving
solve problems that arise in mathematics and in other contexts

Mathworld.com Classification:
Number Theory > Arithmetic > Fractions > Fraction

- Spinners A and B are spun. On each spinner, the arrow is equally likely to land on each number. What is the probability that the product of the two spinners' numbers is even?



(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

2004 AMC 8, Problem #21
"Write out all of the possible outcomes."

- **Solution**

(D) In eight of the twelve outcomes the product is even: 1×2, 2×1, 2×2, 2×3, 3×2, 4×1, 4×2, 4×3. In four of the twelve, the product is odd: 1×1, 1×3, 3×1, 3×3. So the probability that the product is even is $\frac{8}{12}$ or $\frac{2}{3}$.

OR

To get an odd product, the result of both spins must be odd. The probability of odd is $\frac{1}{2}$ on Spinner A and $\frac{2}{3}$ on Spinner B. So the probability of an odd product is $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{1}{3}$. The probability of an even product, then, is $1 - \frac{1}{3} = \frac{2}{3}$.

Difficulty: Medium-hard

NCTM Standard: Data Analysis and Probability
understand and apply basic concepts of probability

Mathworld.com Classification:
Probability and Statistics > Probability > Probability

- At a party there are only single women and married men with their wives. The probability that a randomly selected woman is single is $\frac{2}{5}$. What fraction of the people in the room are married men?

(A) $\frac{1}{3}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{5}{12}$ **(E)** $\frac{3}{5}$

2004 AMC 8, Problem #22

"Because $\frac{2}{5}$ of all the women in the room are single, there are two single women for every three married women in the room.

- **Solution**

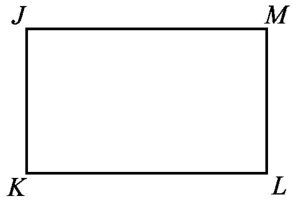
(B) Because $\frac{2}{5}$ of all the women in the room are single, there are two single women for every three married women in the room. There are also two single women for every three married men in the room. So out of every $2+3+3=8$ people, 3 are men. The fraction of the people who are married men is $\frac{3}{8}$.

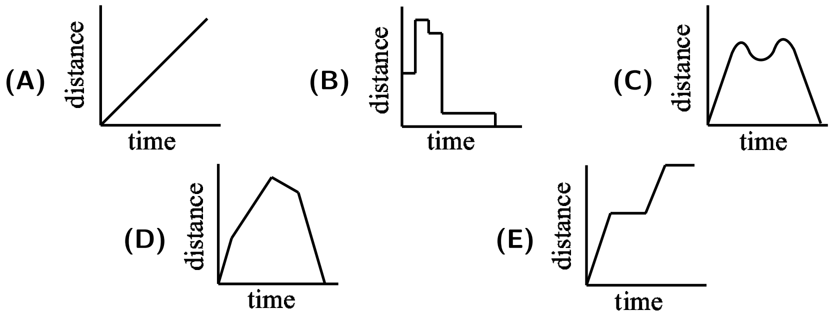
Difficulty: Hard

NCTM Standard: Data Analysis and Probability
understand and apply basic concepts of probability

Mathworld.com Classification:
Probability and Statistics > Probability > Probability

- Tess runs counterclockwise around rectangular block $JKLM$. She lives at corner J. Which graph could represent her straight-line distance from home?





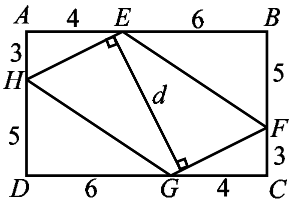
2004 AMC 8, Problem #23
“The distance increases as Tess moves from J to K

- **Solution**

  (D) The distance increases as Tess moves from J to K, and continues at perhaps a different rate as she moves from K to L. The greatest distance from home will occur at L. The distance decreases as she runs from L to M and continues at perhaps a different rate as she moves from M to J. Graph D shows these changes.

Difficulty: Medium-hard
NCTM Standard: Problem Solving
solve problems that arise in mathematics and in other contexts
Mathworld.com Classification:
Applied Mathematics > Data Visualization > Function Graph

- In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



(A) 6.8 **(B)** 7.1 **(C)** 7.6 **(D)** 7.8 **(E)** 8.1

2004 AMC 8, Problem #24
"Use the Pythagorean Theorem to solve for HE"

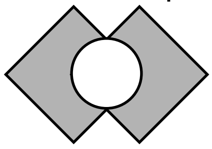
- **Solution (C)** By the Pythagorean Theorem, $HE = 5$. Rectangle $ABCD$ has area $10 \times 8 = 80$, and the corner triangles have areas $\frac{1}{2} \times 3 \times 4 = 6$ and $\frac{1}{2} \times 6 \times 5 = 15$. So the area of $EFGH$ is $80 - (2)(6) - (2)(15) = 38$. Because the area of $EFGH$ is $EH \times d$ and $EH = 5$, $38 = 5 \times d$, so $d = 7.6$.

Difficulty: Medium-hard

NCTM Standard: Geometry
analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Mathworld.com Classification:
Geometry > Plane Geometry > Quadrilaterals > Parallelogram

- Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?



(A) $16 - 4\pi$ **(B)** $16 - 2\pi$ **(C)** $28 - 4\pi$ **(D)** $28 - 2\pi$ **(E)** $32 - 2\pi$

2004 AMC 8, Problem #25
"Draw in the square that exists in the middle. What is its side length?

- **Solution**

(D) The overlap of the two squares is a smaller square with side length 2, so the area of the region covered by the squares is $2(4\times4)-(2\times2) = 32-4 = 28$. The diameter of the circle has length $\sqrt{2^2+2^2} = \sqrt{8}$, the length of the diagonal of the smaller square. The shaded area created by removing the circle from the squares is $28 - \pi\left(\frac{\sqrt{8}}{2}\right)^2 = 28 - 2\pi$.

Difficulty: Hard

NCTM Standard: Geometry
analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Mathworld.com Classification:
Geometry > Plane Geometry > Circles > Diameter