Aunt Anna is 42 years old. Caitlin is 5 years younger than Brianna, and Brianna is half as old as Aunt Anna. How old is Caitlin?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 21 **(E)** 37

2000 AMC 8, Problem #1— "Brianna is half as old as Aunt Anna who is 42, and Caitlin is 5 years younger than Brianna."

Solution

Answer (B): Brianna is half as old as Aunt Anna, so Brianna is 21 years old. Caitlin is 5 years younger than Brianna, so Caitlin is 16 years old.

Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 6-8: model and solve contextualized problems using various representations, such as graphs, tables, and equations.

Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division > Division
Number Theory > Arithmetic > Addition and Subtraction > Subtraction

Which of these numbers is less than its reciprocal?

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

2000 AMC 8, Problem #2— "Two numbers are reciprocals if and only if their product is 1."

Solution

Answer (A): The number 0 has no reciprocal, and 1 and -1 are their own reciprocals. This leaves only 2 and -2. The reciprocal of 2 is $\frac{1}{2}$, but 2 is not less than $\frac{1}{2}$. The reciprocal of -2 is $-\frac{1}{2}$, and -2 is less than $-\frac{1}{2}$.

Difficulty: Medium

NCTM Standard: Number and Operations Standard for Grades 6-8: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division > Reciprocal

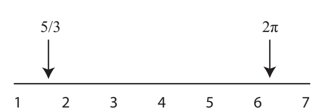
How many whole numbers lie in the interval between $\frac{5}{3}$ and 2π ?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** infinitely many

2000 AMC 8, Problem #3—
"$\frac{5}{3}$ is greater than 1 and less than 2; 2π is greater than 6 and less than 7."

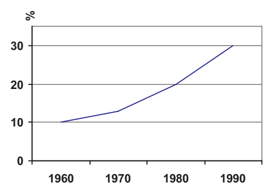
Solution

Answer (D): The smallest whole number in the interval is 2 because $\frac{5}{3}$ is more than 1 but less than 2. The largest whole number in the interval is 6 because 2π is more than 6 but less than 7. There are five whole numbers in the interval. They are 2, 3, 4, 5, and 6.

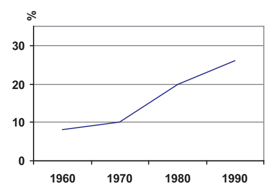


Difficulty: Medium-hard
NCTM Standard: Number and Operations Standard for Grades 6-8: compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line.
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Fraction
Number Theory > Constants > Pi

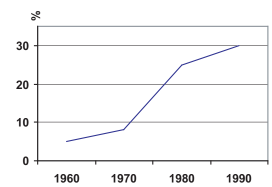
In 1960 only 5% of the working adults in Carlin City worked at home. By 1970 the "at-home" work force had increased to 8%. In 1980 there were approximately 15% working at home, and in 1990 there were 30%. The graph that best illustrates this is:



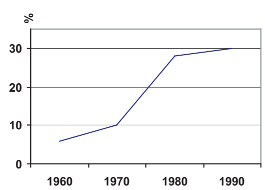
(A)



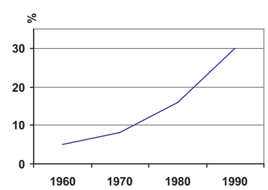
(B)



(C)



(D)



(E)

2000 AMC 8, Problem #4— "The data are 1960(5%), 1970(8%), 1980(15%), and 1990(30%)."

Solution

Answer (E): The data are 1960(5%), 1970(8%), 1980(15%), and 1990(30%). Only graph (E) has these entries.

Difficulty: Easy

NCTM Standard: Geometry Standard for Grades 68: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

Each principal of Lincoln High School serves exactly one 3-year term. What is the maximum number of principals this school could have during an 8-year period?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 8

2000 AMC 8, Problem #5— "The first year of the 8-year has to be the final year of a principal's term."

Solution

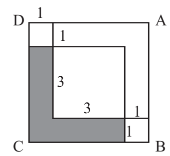
Answer (C): If the first year of the 8-year period was the final year of a principal's term, then in the next six years two more principals would serve, and the last year of the period would be the first year of the fourth principal's term. Therefore, the maximum number of principals who can serve during an 8-year period is 4.

Difficulty: Hard

NCTM Standard: Algebra Standard for Grades 68 : model and solve contextualized problems using various representations, such as graphs, tables, and equations.

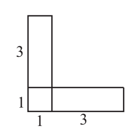
Mathworld.com Classification: Calculus and Analysis > Functions > Period
Calculus and Analysis > Calculus > Maxima and Minima > Maximum

Figure $ABCD$ is a square. Inside this square three smaller squares are drawn with side lengths as labeled. the area of the shaded L-shaped region is



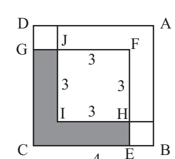
(A) 7 **(B)** 10 **(C)** 12.5 **(D)** 14 **(E)** 15

2000 AMC 8, Problem #6— "Look at the shade area as two rectangles."



Solution

Answer (A): The L-shaped region is made up of two rectangles with area $3 \times 1 = 3$ plus the corner square with area $1 \times 1 = 1$, so the area of the L-shaped figure is $2 \times 3 + 1 = 7$.

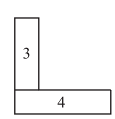


OR

Square $FECG-$ square $FHIJ = 4 \times 4 - 3 \times 3 = 16 - 9 = 7$.

OR

The L-shaped region can be decomposed into a 4×1 rectangle and a 3×1 rectangle. So the total area is 7.



Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 68: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

What is the minimum possible product of three different numbers of the set $\{-8, -6, -4, 0, 3, 5, 7\}$?

(A) -336 **(B)** -280 **(C)** -210 **(D)** -192 **(E)** 0

2000 AMC 8, Problem #7—

"The only way to get a negative product using three numbers is to multiply one negative number and two positives or three negatives."

Solution

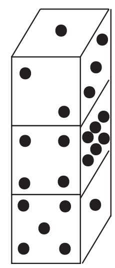
Answer (B): The only way to get a negative product using three numbers is to multiply one negative number and two positives or three negatives. Only two reasonable choices exist: $(-8)\times(-6)\times(-4) = (-8)\times(24) = -192$ and $(-8)\times 5\times 7 = (-8)\times 35 = -280$. The latter is smaller.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 68: use factors, multiples, prime factorization, and relatively prime numbers to solve problems.

Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division

Three dice with faces numbered 1 through 6 are stacked as shown. Seven of the eighteen faces are visible, leaving eleven faces hidden(back, bottom, between). The total number of dots NOT visible in this view is



(A) 21 **(B)** 22 **(C)** 31 **(D)** 41 **(E)** 53

2000 AMC 8, Problem #8—

"The total number of dots that are not visible equals total dots minus visible dots."

Solution

Answer (D): The numbers on one die total $1+2+3+4+5+6 = 21$, so the numbers on the three dice total 63. Numbers 1, 1, 2, 3, 4, 5, 6 are visible, and these total 22. This leaves $63 - 22 = 41$ not seen.

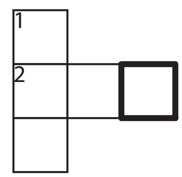
Difficulty: Medium-easy

NCTM Standard: Problem Solving Standard for Grades 6-8: solve problems that arise in mathematics and in other contexts

Mathworld.com Classification: Recreational Mathematics > Games > Dice Games > Dice

Three-digit powers of 2 and 5 are used in this *cross-number* puzzle. What is the only possible digit for the outlined square?

ACROSS	DOWN
2. 2^m	**1.** 5^n



(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2000 AMC 8, Problem #9—
"The 3-digit powers of 5 are 125 and 625, so space 2 is filled with a 2."

Solution

Answer (D): The 3-digit powers of 5 are 125 and 625, so space 2 is filled with a 2. The only 3-digit power of 2 beginning with 2 is 256, so the outlined block is filled with a 6.

Difficulty: Medium-hard

NCTM Standard: Problem Solving Standard for Grades 6-8: solve problems that arise in mathematics and in other contexts

Mathworld.com Classification: Recreational Mathematics > Puzzles

Ara and Shea were once the same height. Since then Shea has grown 20% while Ara has grown half as many inches as Shea. Shea is now 60 inches tall. How tall, in inches, is Ara now?

(A) 48 **(B)** 51 **(C)** 52 **(D)** 54 **(E)** 55

2000 AMC 8, Problem #10— "The starting height was $\frac{60}{1.2} = 50$ inches."

Solution

Answer (E): Shea is 60 inches tall. This is 1.2 times the common starting height, so the starting height was $\frac{60}{1.2} = 50$ inches. Shea has grown $60 - 50 = 10$ inches. Therefore, Ara grew 5 inches and is now 55 inches tall.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 68: develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Proportional

The number 64 has the property that it is divisible by its units digit. How many whole numbers between 10 and 50 have this property?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 20

2000 AMC 8, Problem #11— "All the numbers ending with 1, 2, or 5 have this property."

Solution

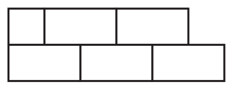
Answer (C): Twelve numbers ending with 1, 2, or 5 have this property. They are 11, 12, 15, 21, 22, 25, 31, 32, 35, 41, 42, and 35. In addition, we have 33, 24, 44, 36, and 48, for a total of 17. (Note that 20, 30, and 40 are not divisible by 0, since division by 0 is not defined.)

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 68: understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Mathworld.com Classification: Number Theory > Rational Numbers > Digit

A block wall 100 feet long and 7 feet high will be constructed using blocks that are 1 foot high and either 2 feet long or 1 foot long (no blocks may be cut). The vertical joins in the blocks must be staggered as shown, and the wall must be even on the ends. What is the smallest number of blocks needed to build this wall?



(A) 344 **(B)** 347 **(C)** 350 **(D)** 353 **(E)** 356

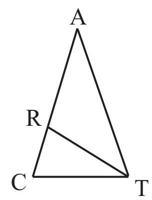
2000 AMC 8, Problem #12—
"To stagger the joins, we need only to replace, in every other row, one of the longer blocks by two shorter ones, placing one at each end."

Solution

Answer (D): If the vertical joins were not staggered, the wall could be build with $\frac{1}{2}(100 \times 7) = 350$ of the two-foot blocks. To stagger the joins, we need only to replace, in every other row, one of the longer blocks by two shorter ones, placing one at each end. To minimize the number of blocks this should be done in rows 2, 4, and 6. This adds 3 blocks to the 350, making a total of 353.

Difficulty: Hard
NCTM Standard: Problem Solving Standard for Grades 68: apply and adapt a variety of appropriate strategies to solve problems.
Mathworld.com Classification: Geometry > General Geometry

In triangle CAT, we have $\angle ACT = \angle ATC$ and $\angle CAT = 36°$. If $\overline{TR}$ bisects $\angle ATC$, then $\angle CRT =$



(A) $16°$ **(B)** $51°$ **(C)** $72°$ **(D)** $90°$ **(E)** $108°$

2000 AMC 8, Problem #13—
"$2(\angle ATC) = 180° - 36° = 144°$ and $\angle ATC = \angle ACT = 72°$."

Solution

Answer (C): Since $\angle ACT = \angle ATC$ and $\angle CAT = 36°$, we have $2(\angle ATC) = 180° - 36° = 144°$ and $\angle ATC = \angle ACT = 72°$. Because $\overline{TR}$ bisects $\angle ATC$, $\angle CTR = \frac{1}{2}(72°) = 36°$. In triangle CRT, $\angle CRT = 180° - 36° - 72° = 72°$.
Note that some texts use $\angle ACT$ to define the angle and $m\angle ACT$ to indicate its measure.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 68: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Trigonometry > Angles

What is the units digit of $19^{19} + 99^{99}$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 8 **(E)** 9

2000 AMC 8, Problem #14— "The units digit of a power of an integer is determined by the units digit of the integer."

Solution

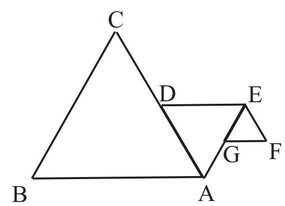
Answer (D): The units digit of a power of an integer is determined by the units digit of the integer; that is, the tens digit, hundreds digit, etc... of the integer have no effect on the units digit of the result. In this problem, the units digit of 19^{19} is the units digit of 9^{19}. Note that $9^1 = 9$ ends in 9, $9^2 = 81$ ends in 1, $9^3 = 729$ ends in 9, and, in general, the units digit of odd powers of 9 is 9, whereas the units digit of even powers of 9 is 1. Since both exponents are odd, the sum of their units digits is $9 + 9 = 18$, the units digit of which is 8.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 68: Understand meanings of operations and how they relate to one another.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Powers

Triangle ABC, ADE, and EFG are all equilateral. Points D and G are midpoints of $\overline{AC}$ and $\overline{AE}$, respectively. If $AB = 4$, what is the perimeter of figure $ABCDEFG$?



(A) 12 **(B)** 13 **(C)** 15 **(D)** 18 **(E)** 21

2000 AMC 8, Problem #15—
"$AB = BC = 4$**,** $CD = DE = 2$**,** $EF + FG + GA = 1$**."**

Solution

Answer (C): We have

$$AB + BC + CD + DE + EF + FG + GA =$$
$$4 + 4 + 2 + 2 + 1 + 1 + 1 = 15$$

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 68: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Calculus and Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter

In order for Mateen to walk a kilometer(1000m) in his rectangular backyard, he must walk the length 25 times or walk its perimeter 10 times. What is the area of Mateen's backyard in square meters?

(A) 40 **(B)** 200 **(C)** 400 **(D)** 500 **(E)** 1000

2000 AMC 8, Problem #16— "Perimeter is two lengths and two widths for a rectangle."

Solution

Answer (C): The perimeter is $1000 \div 10 = 100$, and this is two lengths and two widths. The length of the backyard is $1000 \div 25 = 40$. Since two lengths total 80, the two widths total 20, and the width is 10. The area is $10 \times 40 = 400$.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 68: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Calculus and Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter
Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

The operation $\otimes$ is defined for all nonzero numbers by $a \otimes b = \frac{a^2}{b}$. Determine $[(1 \otimes 2) \otimes 3] - [1 \otimes (2 \otimes 3)]$.

(A) $-\frac{2}{3}$ **(B)** $-\frac{1}{4}$ **(C)** 0 **(D)** $\frac{1}{4}$ **(E)** $\frac{2}{3}$

2000 AMC 8, Problem #17— "Follow the operation order."

Solution

Answer (A): We have

$$(1 \otimes 2) \otimes 3 = \frac{1^2}{2} \otimes 3 = \frac{1}{2} \otimes 3 = \frac{(\frac{1}{2})^2}{3} = \frac{\frac{1}{4}}{3} = \frac{1}{12},$$

and

$$1 \otimes (2 \otimes 3) = 1 \otimes (\frac{2^2}{3}) = 1 \otimes \frac{4}{3} = \frac{1^2}{\frac{4}{3}} = \frac{3}{4}.$$

Therefore,

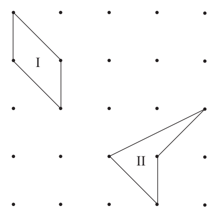
$$(1 \otimes 2) \otimes 3 - 1 \otimes (2 \otimes 3) = \frac{1}{12} - \frac{3}{4} = \frac{1}{12} - \frac{9}{12} = -\frac{8}{12} = -\frac{2}{3}.$$

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 68: understand meanings of operations and how they relate to one another.

Mathworld.com Classification: Calculus and Analysis > Functions > Operation

Consider these two geoboard quadrilaterals. Which of the following statements is true?



(A) The area of quadrilateral I is more than the area of quadrilateral II.

(B) The area of quadrilateral I is less than the area of quadrilateral II.

(C) The quadrilaterals have the same area and the same perimeter.

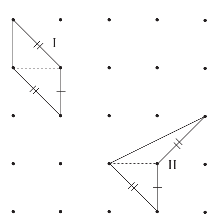
(D) The quadrilaterals have the same area, but the perimeter of I is more than the perimeter of II.

(E) The quadrilaterals have the same area, but the perimeter of I is less than the perimeter of II.

2000 AMC 8, Problem #18—
"Divide each quadrilateral into small triangles and compare."

Solution

Answer (E): Divide each quadrilateral as shown. The resulting triangles each have base 1, altitude 1, and area $\frac{1}{2}$, so the quadrilaterals each have area 1.



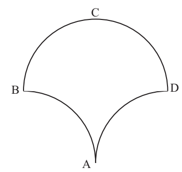
Three sides of quadrilateral I match those of quadrilateral II as indicated by matching marks. The fourth side of quadrilateral I is less than the fourth side of quadrilateral II, hence its perimeter is less, and choice (E) is correct.

Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 68: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area Calculus and Analysis > Differential Geometry > Differential Geometry of Curves > Perimeter

Three circular arcs of radius 5 units bound the region shown. Arcs AB and AD are quarter-circles, and arc BCD is a semicircle. What is the area, in square units, of the region?

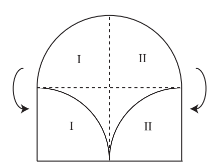


(A) 25 **(B)** $10 + 5\pi$ **(C)** 50 **(D)** $50 + 5\pi$ **(E)** 25π

2000 AMC 8, Problem #19— "Change the shape of the figure to a rectangle."

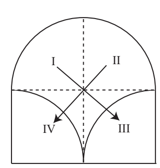
Solution

Answer (C): Divide the semicircle in half and rotate each half down to fill the space below the quarter-circles. The figure formed is a rectangle with dimensions 5 and 10. The area is 50.



OR

Slide I into III and II into IV as indicated by the arrows to create the 5×10 rectangle.



Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 68: apply transformations and use symmetry to analyze mathematical situations.

Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

You have nine coins: a collection of pennies, nickels, dimes, and quarters having a total value of \$1.02, with at least one coin of each type. How many dimes must you have?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2000 AMC 8, Problem #20—
"Since the total value is \$1.02, you must have either 2 or 7 pennies."

Solution

Answer (A): Since the total value is \$1.02, you must have either 2 or 7 pennies. It is impossible to have 7 pennies, since the two remaining coins cannot have a value of 95 cents. With 2 pennies the remaining 7 coins have a value of \$1.00. Either 2 or 3 of these must be quarters. If you have 2 quarters, the other 5 coins would be dimes, and you would have no nickels. The only possible solution is 3 quarters, 1 dime, 3 nickels and 2 pennies.

Difficulty: Medium-hard

NCTM Standard: Algebra Standard for Grades 68: relate and compare different forms of representation for a relationship.

Mathworld.com Classification: Number Theory > Arithmetic > Addition and Subtraction

Keiko tosses one penny and Ephraim tosses two pennies. The probability that Ephraim gets the same number of heads that Keiko gets is

(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

2000 AMC 8, Problem #21— "Make a complete list of equally likely outcomes."

Solution

Answer (B): Make a complete list of equally likely outcomes:

Keiko	Ephraim	Same Number of Heads?
H	HH	No
H	HT	Yes
H	TH	Yes
H	TT	No
T	HH	No
T	HT	No
T	TH	No
T	TT	Yes

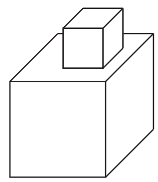
The probability that they have the same number of heads is $\frac{3}{8}$.

Difficulty: Hard

NCTM Standard: Data Analysis and Probability Standard for Grades 68: compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

Mathworld.com Classification: Probability and Statistics > Probability

A cube has edge length 2. Suppose that we glue a cube of edge length 1 on top of the big cube so that one of its faces rests entirely on the top face of the larger cube. The percent increase in the surface area (sides, top, and bottom) from the original cube to the new solid formed is closest to:



(A) 10 **(B)** 15 **(C)** 17 **(D)** 21 **(E)** 25

2000 AMC 8, Problem #22—
"How much area on big cube is covered, how much area on little cube is exposed."

Solution

Answer (C): The area of each face of the larger cube is $2^2 = 4$. There are six faces of the cube, so its surface area is $6(4) = 24$. When we add the smaller cube, we decrease the original surface area by 1, but we add $5(1^2) = 5$ units of area (1 unit for each of the five unglued faces of the smaller cube), This is a net increase of 4 from the original surface area, and 4 is $\frac{4}{24} = \frac{1}{6} \approx 16.7\%$ of 24. The closest value given is 17.

Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 68: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Calculus and Analysis > Differential Geometry > Differential Geometry of Surfaces > Surface Area

There is a list of seven numbers. The average of the first four numbers is 5, and the average of the last four numbers is 8. If the average of all seven numbers is $6\frac{4}{7}$, then the number common to both sets of four numbers is

(A) $5\dfrac{3}{7}$ **(B)** 6 **(C)** $6\dfrac{4}{7}$ **(D)** 7 **(E)** $7\dfrac{3}{7}$

2000 AMC 8, Problem #23—
"The fourth number = (sum of the first four numbers + sum of the last four numbers) − sum of all seven numbers."

Solution

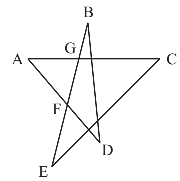
Answer (B): Since the average of all seven numbers is $6\frac{4}{7} = \frac{46}{7}$, the sum of the seven numbers is $7 \times \frac{46}{7} = 46$. The sum of the first four numbers is $4 \times 5 = 20$ and the sum of the last four numbers is $4 \times 8 = 32$. Since the fourth number is used in each of these two sums, the fourth number must be $(20 + 32) - 46 = 6$.

Difficulty: Medium-hard

NCTM Standard: Algebra Standard for Grades 68: use mathematical models to represent and understand quantitative relationships.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Means

If $\angle A = 20^\circ$ and $\angle AFG = \angle AGF$, Then $\angle B + \angle D =$

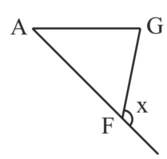


(A) 48° **(B)** 60° **(C)** 72° **(D)** 80° **(E)** 90°

2000 AMC 8, Problem #24— "$\angle AFG = 80^\circ$, and $\angle BFD = 100^\circ$."

Solution

Answer (D): Since $\angle AFG = \angle AGF$ and $\angle GAF + \angle AFG + \angle AGF = 180^\circ$, we have $20^\circ + 2(\angle AFG) = 180^\circ$. So $\angle AFG = 80^\circ$. Also, $\angle AFG + \angle BFD = 190^\circ$, so $\angle BFD = 100^\circ$. The sum of the angles of $\triangle BFD$ is 180°, so $\angle B + \angle D = 80^\circ$.



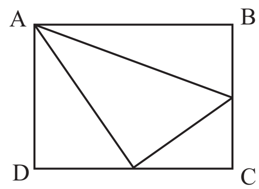
Note: In $\triangle AFG, \angle AFG = \angle B + \angle D$. In general, an exterior angle of a triangle equals the sum of its remote interior angles. For example, in $\triangle GAF, \angle x = \angle GAF + \angle AGF$. Note that, as in Problem 13, some texts use different symbols to represent an angle and its degree measure.

Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 68: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Trigonometry > Angles

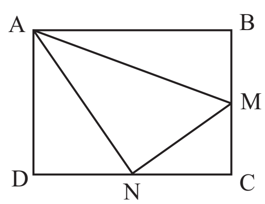
The area of rectangle $ABCD$ is 72. If point A and the midpoints of $\overline{BC}$ and $\overline{CD}$ are joined to form a triangle, the area of that triangle is



(A) 21 **(B)** 27 **(C)** 30 **(D)** 36 **(E)** 40

2000 AMC 8, Problem #25—
"The three right triangles lie outside $\triangle AMN$ have areas: $\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{8}$."

Solution



Answer (B): Three right triangles lie outside $\triangle AMN$. Their areas are $\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{8}$ for a total of $\frac{5}{8}$ of the rectangle. The area of $\triangle AMN$ is $\frac{3}{8}(72) = 27$.

OR

Let the rectangle have sides of $2a$ and $2b$ so that $4ab = 72$ and $ab = 18$.Three right triangles lie outside triangle AMN, and their areas are $\frac{1}{2}(2a)(b)$, $\frac{1}{2}(2b)(a)$, $\frac{1}{2}(a)(b)$, for a total of $\frac{5}{2}(ab) = \frac{5}{2}(18) = 45$. The area of triangle AMN is $72 - 45 = 27$.

Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 68: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area