



## AMERICAN MATHEMATICS COMPETITIONS

15th ANNUAL

# AMC → 8

(formerly AJHSME)

## TUESDAY, NOVEMBER 16, 1999

Sponsored by

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### INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC → 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, ruler, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working the problems. You will have **40 MINUTES** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

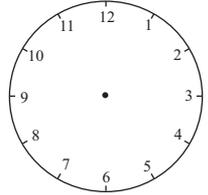
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1.  $(6 ? 3) + 4 - (2 - 1) = 5$ . To make this statement true, the question mark between the 6 and the 3 should be replaced by

(A)  $\div$       (B)  $\times$       (C)  $+$       (D)  $-$       (E) None of these

2. What is the degree measure of the smaller angle formed by the hands of a clock at 10 o'clock?

(A) 30      (B) 45      (C) 60  
(D) 75      (E) 90

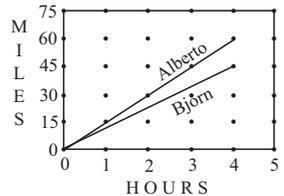


3. Which triplet of numbers has a sum NOT equal to 1?

(A)  $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$       (B)  $(2, -2, 1)$       (C)  $(0.1, 0.3, 0.6)$   
(D)  $(1.1, -2.1, 1.0)$       (E)  $(-\frac{3}{2}, -\frac{5}{2}, 5)$

4. The diagram shows the miles traveled by bikers Alberto and Bjorn. After four hours about how many more miles has Alberto biked than Bjorn?

(A) 15      (B) 20      (C) 25  
(D) 30      (E) 35



5. A rectangular garden 50 feet long and 10 feet wide is enclosed by a fence. To make the garden larger, while using the same fence, its shape is changed to a square. By how many square feet does this enlarge the garden?

(A) 100      (B) 200      (C) 300      (D) 400      (E) 500

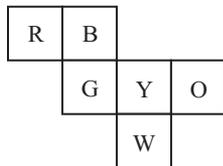
6. Bo, Coe, Flo, Jo, and Moe have different amounts of money. Neither Jo nor Bo has as much money as Flo. Both Bo and Coe have more than Moe. Jo has more than Moe, but less than Bo. Who has the least amount of money?

(A) Bo      (B) Coe      (C) Flo      (D) Jo      (E) Moe

7. The third exit on a highway is located at milepost 40 and the tenth exit is at milepost 160. There is a service center on the highway located three-fourths of the way from the third exit to the tenth exit. At what milepost would you expect to find this service center?

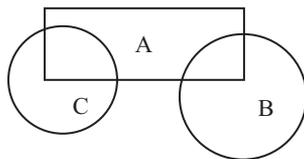
(A) 90      (B) 100      (C) 110      (D) 120      (E) 130

8. Six squares are colored, front and back, (R=red, B=blue, O=orange, Y=yellow, G=green, and W=white). They are hinged together as shown, then folded to form a cube. The face opposite the white face is



(A) B      (B) G      (C) O      (D) R      (E) Y

9. Three flower beds overlap as shown. Bed A has 500 plants, bed B has 450 plants, and bed C has 350 plants. Beds A and B share 50 plants, while beds A and C share 100. The total number of plants is

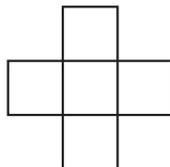


(A) 850      (B) 1000      (C) 1150      (D) 1300  
(E) 1450

10. A complete cycle of a traffic light takes 60 seconds. During each cycle the light is green for 25 seconds, yellow for 5 seconds, and red for 30 seconds. At a randomly chosen time, what is the probability that the light will NOT be green?

(A)  $\frac{1}{4}$       (B)  $\frac{1}{3}$       (C)  $\frac{5}{12}$       (D)  $\frac{1}{2}$       (E)  $\frac{7}{12}$

11. Each of the five numbers 1,4,7,10, and 13 is placed in one of the five squares so that the sum of the three numbers in the horizontal row equals the sum of the three numbers in the vertical column. The largest possible value for the horizontal or vertical sum is



(A) 20      (B) 21      (C) 22      (D) 24      (E) 30

12. The ratio of the number of games won to the number of games lost (no ties) by the Middle School Middies is  $11/4$ . To the nearest whole percent, what percent of its games did the team lose?

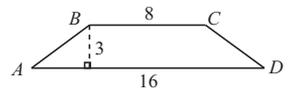
(A) 24      (B) 27      (C) 36      (D) 45      (E) 73

13. The average age of the 40 members of a computer science camp is 17 years. There are 20 girls, 15 boys, and 5 adults. If the average age of the girls is 15 and the average age of the boys is 16, what is the average age of the adults?

(A) 26      (B) 27      (C) 28      (D) 29      (E) 30

14. In trapezoid  $ABCD$ , the side  $AB$  and  $CD$  are equal. The perimeter of  $ABCD$  is

(A) 27      (B) 30      (C) 32  
(D) 34      (E) 48



15. Bicycle license plates in Flatville each contain three letters. The first is chosen from the set  $\{C,H,L,P,R\}$ , the second from  $\{A,I,O\}$ , and the third from  $\{D,M,N,T\}$ .



When Flatville needed more license plates, they added two new letters. The new letters may both be added to one set or one letter may be added to one set and one to another set. What is the largest possible number of ADDITIONAL license plates than can be made by adding two letters?

(A) 24      (B) 30      (C) 36      (D) 40      (E) 60

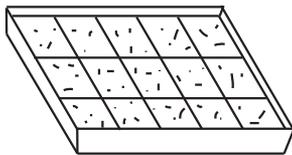
16. Tori's mathematics test had 75 problems: 10 arithmetic, 30 algebra, and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of the algebra, and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% of the problems right. How many more questions would she have needed to answer correctly to earn a 60% passing grade?

(A) 1      (B) 5      (C) 7      (D) 9      (E) 11

Problems 17, 18, and 19 refer to the following:

Cookies For a Crowd

At Central Middle School the 108 students who take the AMC → 8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie’s Best Bar Cookies this year. Their recipe, which makes a pan of 15 cookies, list these items:



$1\frac{1}{2}$  cups of flour, 2 eggs, 3 tablespoons butter,  $\frac{3}{4}$  cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipe.

- 17. Walter can buy eggs by the half-dozen. How many half-dozens should he buy to make enough cookies? (Some eggs and some cookies may be left over.)  
 (A) 1    (B) 2    (C) 5    (D) 7    (E) 15
- 18. They learn that a big concert is scheduled for the same night and attendance will be down 25%. How many recipes of cookies should they make for their smaller party?  
 (A) 6    (B) 8    (C) 9    (D) 10    (E) 11
- 19. The drummer gets sick. The concert is cancelled. Walter and Gretel must make enough pans of cookies to supply 216 cookies. There are 8 tablespoons in a stick of butter. How many sticks of butter will be needed? (Some butter may be left over, of course.)  
 (A) 5    (B) 6    (C) 7    (D) 8    (E) 9

- 20. Figure 1 is called a "stack map." The numbers tell how many cubes are stacked in each position. Fig. 2 shows these cubes, and Fig. 3 shows the view of the stacked cubes as seen from the front.



Figure 1

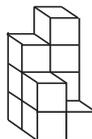


Figure 2

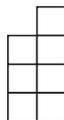


Figure 3

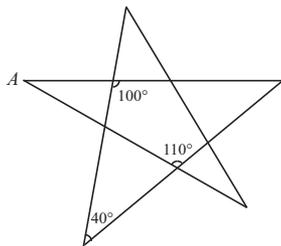


Figure 4

- (A)
- (B)
- (C)
- (D)
- (E)

21. The degree measure of angle  $A$  is

- (A) 20    (B) 30    (C) 35    (D) 40    (E) 45

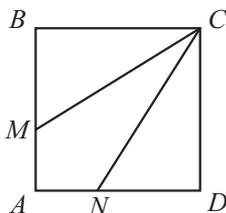


22. In a far-off land three fish can be traded for two loaves of bread and a loaf of bread can be traded for four bags of rice. How many bags of rice is one fish worth?

- (A)  $\frac{3}{8}$     (B)  $\frac{1}{2}$     (C)  $\frac{3}{4}$     (D)  $2\frac{2}{3}$     (E)  $3\frac{1}{3}$

23. Square  $ABCD$  has sides of length 3. Segments  $CM$  and  $CN$  divide the square's area into three equal parts. How long is segment  $CM$ ?

- (A)  $\sqrt{10}$     (B)  $\sqrt{12}$     (C)  $\sqrt{13}$   
 (D)  $\sqrt{14}$     (E)  $\sqrt{15}$

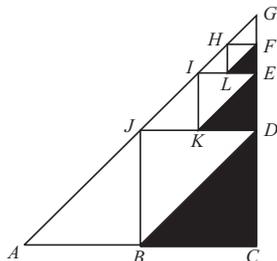


24. When  $1999^{2000}$  is divided by 5, the remainder is

- (A) 4    (B) 3    (C) 2    (D) 1    (E) 0

25. Points  $B$ ,  $D$ , and  $J$  are midpoints of the sides of right triangle  $ACG$ . Points  $K$ ,  $E$ ,  $I$  are midpoints of the sides of triangle  $JDG$ , etc. If the dividing and shading process is done 100 times (the first three are shown) and  $AC = CG = 6$ , then the total area of the shaded triangles is nearest

- (A) 6    (B) 7    (C) 8    (D) 9    (E) 10



**SOLUTIONS**

Your School Manager will be sent at least one copy of the 1999 AMC  $\rightarrow$  8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

**WRITE TO US!**

Comments about the problems and solutions for this AMC  $\rightarrow$  8 should be addressed to:

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Lincoln, NE 68588-0658 Phone: 402-472-2257; Fax: 402-472-6087

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(formerly AHSME)

The AMC  $\rightarrow$  10 and AMC  $\rightarrow$  12 are 25-question, 75-minute contests with 5 choices of answers for each problem (A through E). Schools with high scoring students on the AMC  $\rightarrow$  8 will receive an Invitation Brochure for the 2000 AMC  $\rightarrow$  10. The best way to prepare for these upper level contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

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Each price is for an exam and its solutions for one year. Specify the years you want and how many copies of each exam. All prices effective to September 1, 2000.

AMC  $\rightarrow$  8 (Junior High Exam), 1985-99, \$1 per copy per year.

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Books (Exams and Solutions):

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1999



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**DO NOT OPEN UNTIL  
TUESDAY, NOVEMBER 16, 1999**

**\*\* Administration On An Earlier Date Will Disqualify Your School's Results \*\***

1. All information (Rules and Instructions) needed to administer this exam is contained in the **TEACHERS' MANUAL**, which is outside of this package. **PLEASE READ THE MANUAL BEFORE NOVEMBER 16, 1999.** Nothing is needed from inside this package until November 16.
2. Your **PRINCIPAL** or **VICE-PRINCIPAL** must verify on the **AMC → 8 CERTIFICATION Form** that all rules associated with the conduct of the exam were followed.
3. The Answer Forms must be mailed **First Class** to the **AMC office** no later than 24 hours following the exam.
4. **THE AMC → 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.**
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