$(6\ ?\ 3) + 4 - (2 - 1) = 5$. To make this statement true, the question mark between the 6 and the 3 should be replaced by

(A) $\div$ **(B)** $\times$ **(C)** $+$ **(D)** $-$ **(E)** None of these

1999 AMC 8, Problem #1— "Simplify first, to find what $(6\ ?\ 3)$ should equal."

Solution

Answer (A):

$$
\begin{aligned}
&(6\ ?\ 3) + 4 - (2 - 1) = 5 && \\
&(6\ ?\ 3) + 4 - 1 = 5 && \text{(subtract:} 2 - 1 = 1) \\
&(6\ ?\ 3) + 3 = 5 && \text{(subtract:} 4 - 1 = 3) \\
&(6\ ?\ 3) = 2 && \text{(subtract 3 from both sides)} \\
&(6 \div 3) = 2 &&
\end{aligned}
$$

The other operations produce the following result:

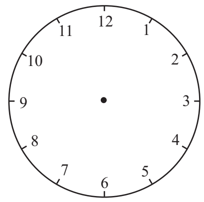
$$
\begin{aligned}
(6 + 3) + 4 - (2 - 1) &= 9 + 4 - 1 = 12 \\
(6 - 3) + 4 - (2 - 1) &= 3 + 4 - 1 = 6 \\
(6 \times 3) + 4 - (2 - 1) &= 18 + 4 - 1 = 21
\end{aligned}
$$

Difficulty: Easy

NCTM Standard: Number and Operations Standard for Grades 6-8: Understand meanings of operations and how they relate to one another.

Mathworld.com Classification: Number Theory > Arithmetic > Addition and Subtraction > Subtraction

What is the degree measure of the smaller angle formed by the hands of a clock at 10 o'clock?

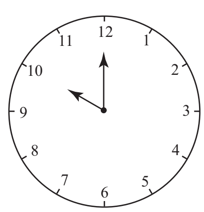


(A) 30 **(B)** 45 **(C)** 60 **(D)** 75 **(E)** 90

1999 AMC 8, Problem #2—
"Find out how many degrees each of the twelve spaces on a clock measures."

Solution

Answer (C): There are $360°$(degrees) in a circle and twelve spaces on a clock. This means that each space measures $30°$. At 10 o'clock the hands point to 10 and 12. They are two spaces or $60°$ apart.



Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6-8: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.

Mathworld.com Classification: Geometry > Trigonometry > Angles

Which triplet of numbers has a sum NOT equal to 1 ?

(A) $(\frac{1}{2}, \frac{1}{3}, \frac{1}{6})$ **(B)** $(2, -2, 1)$ **(C)** $(0.1, 0.3, 0.6)$ **(D)** $(1.1, -2.1, 1.0)$ **(E)** $(-\frac{3}{2}, -\frac{5}{2}, 5)$

1999 AMC 8, Problem #3— "Find the sum of all triplets."

Solution

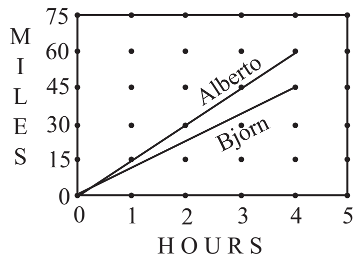
Answer (D): $1.1 + (-2.1) + 1.0 = 0$. The other triplets add to 1 .

Difficulty: Medium-easy

NCTM Standard: Number and Operations Standard for Grades 6-8: select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions
Number Theory > Arithmetic > Addition and Subtraction > Addition

The diagram shows the miles traveled by bikers Alberto and Bjorn. After four hours about how many more miles has Alberto biked then Bjorn?



(A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

1999 AMC 8, Problem #4— "After four hours, how many miles do Alberto and Bjorn each bike?"

Solution

Answer (A): Four hours after starting, Alberto has gone about 60 miles and Bjorn has gone about 45 miles. Therefore, Alberto has biked about 15 more miles.

Difficulty: Easy

NCTM Standard: Data Analysis and Probability Standard for Grades 6-8: discuss and understand the correspondence between data sets and their graphical representations.

Mathworld.com Classification: History and Terminology > Terminology > Diagram

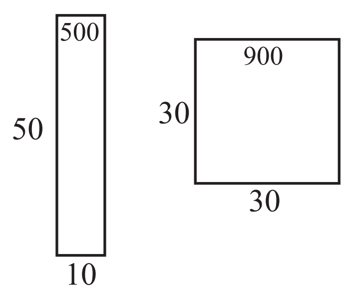
A rectangular garden 50 feet long and 10 feet wide is enclosed by a fence. To make the garden larger, while using the same fence, its shape is changed to a square. By how many square feet does this enlarge the garden?

(A) 100 **(B)** 200 **(C)** 300 **(D)** 400 **(E)** 500

1999 AMC 8, Problem #5—
"The square will have the same perimeter as the rectangular garden."

Solution

Answer (D): The area of the garden was 500 square feet(50×10) and its perimeter was 120 feet, $2 \times (50 + 10)$. The square garden is also enclosed by 120 feet of fence so its sides are each 30 feet long. The square garden's area is 900 square feet (30×30). and this has increased the garden area by 400 square feet.



Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6-8: draw geometric objects with specified properties, such as side lengths or angle measures.

Mathworld.com Classification: Geometry > Plane Geometry > Squares
Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

Bo, Coe, Flo, Jo, and Moe have different amounts of money. Neither Jo nor Bo has as much money as Flo. Both Bo and Coe have more than Moe. Jo has more than Moe, but less than Bo. Who has the least amount of money?

(A) Bo **(B)** Coe **(C)** Flo **(D)** Jo **(E)** Moe

1999 AMC 8, Problem #6— "Cross out the name, that have a greater amount of money."

Solution

Answer (E): From the second sentence, Flo has more than someone so she can't have the least. From the third sentence both Bo and Coe have more than someone so that eliminates them. And, from the fourth sentence, Jo has more than someone, so that leaves only poor Moe!

Difficulty: Easy

NCTM Standard: Problem Solving Standard for Grades 6-8: apply and adapt a variety of appropriate strategies to solve problems.

Mathworld.com Classification: History and Terminology > Terminology > Order

The third exit on a highway is located at milepost 40 and the tenth exit is at milepost 160. There is a service center on the highway located three-fourths of the way from the third exit to the tenth exit. At what milepost would you expect to find this service center?

(A) 90 **(B)** 100 **(C)** 110 **(D)** 120 **(E)** 130

1999 AMC 8, Problem #7—
"The service center is located at a milepost equal to $40 +$ ($\frac{3}{4}$of the difference in mileage between the 3rd and the 10th exit)."

Solution

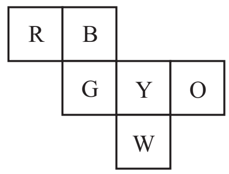
Answer (E): There are $160 - 40 = 120$ miles between the third and tenth exits, so the service center is at milepost $40 + (\frac{3}{4})120 = 40 + 90 = 130$.

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6-8: use geometric models to represent and explain numerical and algebraic relationships.

Mathworld.com Classification: Geometry > Line Geometry > Lines > Real Line

Six squares are colored, front and back, (R=red, B=blue, O=orange, Y=yellow, G=green, and W=white). They are hinged together as shown, then folded to form a cube. The face opposite the white face is

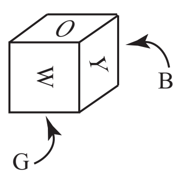


(A) B **(B)** G **(C)** O **(D)** R **(E)** Y

1999 AMC 8, Problem #8—
"Set G as the base, form the cube."

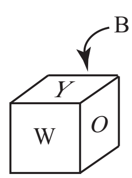
Solution

Answer (A):



When G is arranged to be the base, B is the back face and W is the front face. Thus, B is opposite W.

OR



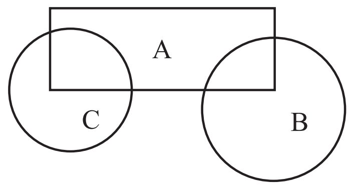
Let Y be the top and fold G, O, and W down. Then B will fold to become the back face and be opposite W.

Difficulty: Medium-easy

NCTM Standard: Geometry Standard for Grades 6-8: use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.

Mathworld.com Classification: Geometry > Solid Geometry > Polyhedra > Cubes

Three flower beds overlap as shown. Bed A has 500 plants, bed B has 450 plants, and bed C has 350 plants. Beds A and B share 50 plants, while beds A and C share 100. The total number of plants is

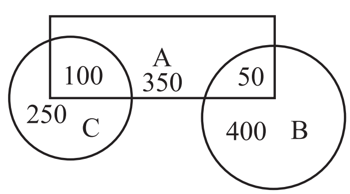


(A) 850 **(B)** 1000 **(C)** 1150 **(D)** 1300 **(E)** 1450

1999 AMC 8, Problem #9—
"Plants shared by two beds have been counted twice."

Solution

Answer (C): Bed A has 350 plants it doesn't share with B or C. Bed B has 400 plants it doesn't share with A or C. And C has 250 it doesn't share with A or B. The total is $350 + 400 + 250 + 50 + 100 = 1150$ plants.



OR

Plants shared by two beds have been counted twice, so the total is $500 + 450 + 350 - 50 - 100 = 1150$.

Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6-8: use geometric models to represent and explain numerical and algebraic relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Addition and Subtraction

A complete cycle of a traffic light takes 60 seconds. During each cycle the light is green for 25 seconds, yellow for 5 seconds, and red for 30 seconds. At a randomly chosen time, what is the probability that the light will NOT be green?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{5}{12}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{7}{12}$

1999 AMC 8, Problem #10—
"The probability of not green $= 1 -$ the probability of green."

Solution

Answer (E):

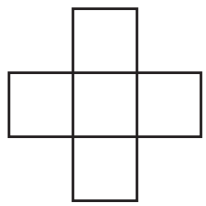
$$\frac{\text{time not green}}{\text{total time}} = \frac{\mathsf{R+Y}}{\mathsf{R+Y+G}} = \frac{35}{60} = \frac{7}{12}.$$

OR

The probability of green is $\frac{25}{60} = \frac{5}{12}$. so the probability of not green is $1 - \frac{5}{12} = \frac{7}{12}$.

Difficulty: Medium
NCTM Standard: Data Analysis and Probability Standard for Grades 6-8: use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
Mathworld.com Classification: Probability and Statistics > Probability

Each of the five numbers 1,4,7,10, and 13 is placed in one of the five squares so that the sum of the three numbers in the horizontal row equals the sum of the three numbers in the vertical column. The largest possible value for the horizontal or vertical sum is

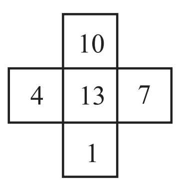


(A) 20 **(B)** 21 **(C)** 22 **(D)** 24 **(E)** 30

1999 AMC 8, Problem #11—
"The sum of horizontal numbers + The sum of vertical numbers = The sum of five numbers + The center number."

Solution

Answer (D): The largest sum occurs when 13 is placed in the center. This sum is $13 + 10 + 1 = 13 + 7 + 4 = 24$. Note: Two other common sums, 18 and 21, are possible.



OR

Since the horizontal sum equals the vertical sum, twice this sum will be the sum of the five numbers plus the number in the center. When the center number is 13, the sum is the largest, $\dfrac{10+4+1+7+2(13)}{2} = \dfrac{48}{2} = 24$. The other four numbers are divided into two pairs with equal sums.

Difficulty: Medium-hard
NCTM Standard: Problem Solving Standard for Grades 6-8: solve problems that arise in mathematics and in other contexts.
Mathworld.com Classification: Algebra > Sums

The ratio of the number of games won to the number of games lost(no ties) by the Middle School Middies is $\frac{11}{4}$. To the nearest whole percent, what percent of its games did the team lose?

(A) 24 **(B)** 27 **(C)** 36 **(D)** 45 **(E)** 73

1999 AMC 8, Problem #12—

"The ratio of the number of games lost to the number of games played is $\frac{4}{11+4}$."

Solution

Answer (B): The Won/Lost ratio is 11/4 so, for some number N, the team won $11N$ games and lost $4N$ games. Thus, the team played $15N$ games and the fraction of games lost is $\dfrac{4N}{15N} = \dfrac{4}{15} \approx 0.27 = 27\%$.

Difficulty: Medium

NCTM Standard: Data Analysis and Probability Standard for Grades 6-8: use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

The average age of the 40 members of a computer science camp is 17 years. There are 20 girls, 15 boys, and 5 adults. If the average age of the girls is 15 and the average age of the boys is 16, what is the average age of the adults?

(A) 26 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 30

1999 AMC 8, Problem #13—
"The sum of the adult's ages = The sum of all ages − (The sum of the girls' ages + The sum of the boys' ages)."

Solution

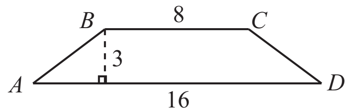
Answer (C): The sum of all ages is $40 \times 17 = 680$. The sum of the girls' ages is $20 \times 15 = 300$ and the sum of the boys' ages is $15 \times 16 = 240$. The sum of the five adults' ages is $680 - 300 - 240 = 140$. Therefore, their average is $\frac{140}{5} = 28$.

Difficulty: Medium-hard

NCTM Standard: Data Analysis and Probability Standard for Grades 6-8: find, use, and interpret measures of center and spread, including mean and interquartile range.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Means > Arithmetic Mean

In trapezoid $ABCD$, the side AB and CD are equal. The perimeter of $ABCD$ is

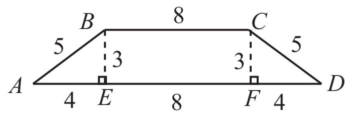


(A) 27 **(B)** 30 **(C)** 32 **(D)** 34 **(E)** 48

1999 AMC 8, Problem #14— "Using the Pythagorean Theorem, $AB^2 = AE^2 + EB^2$."

Solution

Answer (D): When the figure is divided, as shown the unknown sides are the hypotenuses of right triangles with legs of 3 and 4. Using the Pythagorean Theorem yields $AB = CD = 5$. The total perimeter is $16 + 5 + 8 + 5 = 34$.



Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6-8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Quadrilaterals > Isosceles Trapezoid

Bicycle license plates in Flatville each contain three letters. The first is chosen from the set C, H, L, P, R, the second from A, I, O, and the third from D, M, N, T.

When Flatville needed more license plates, they added two new letters. The new letters may both be added to one set or one letter may be added to one set and one to another set. What is the largest possible number of ADDITIONAL license plates than can be made by adding two letters?

(A) 27 **(B)** 30 **(C)** 32 **(D)** 34 **(E)** 48

1999 AMC 8, Problem #15—
"How many license plates could originally be made? Where can the two letters be placed so the most new license plates will be created?"

Solution

Answer (D): Before new letters were added, five different letters could have been chosen for the first position, three for the second, and four for the third. This means that $5 \cdot 3 \cdot 4 = 60$ plates could have been made.
If two letters are added to the second set, then $5 \cdot 5 \cdot 4 = 100$ plates can be made. If one letter is added to each of the second and third sets, then $5 \cdot 4 \cdot 5 = 100$ plates can be made. None of the other four ways to place the two letters will create as many plates. So, $100 - 60 = 40$ ADDITIONAL plates can be made.
Note: Optimum results can usually be obtained in such problems by making the factors as nearly equal as possible.

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard: Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Mathworld.com Classification:
Discrete Mathematics > Combinatorics > Permutations > Combination

Tori's mathematics test had 75 problems: 10 arithmetic, 30 algebra, and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of the algebra, and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% of the problems right. How many more questions would she have needed to answer correctly to earn a 60% passing grade?

(A) 1 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

1999 AMC 8, Problem #16—
"Calculate the total number of questions Tory has answered correctly, and subtract it from $60\%(75)$."

Solution

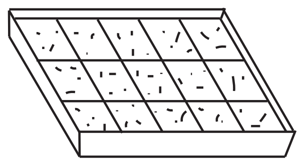
Answer (B): Since $70\%(10)+40\%(30)+60\%(35) = 7+12+21 = 40$, she answered 40 questions correctly. She needed $60\%(75) = 45$ to pass, so she needed 5 mo43 correct answers.

Difficulty: Medium
NCTM Standard: Algebra Standard for Grades 6-8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
Mathworld.com Classification: Number Theory > Arithmetic > Fractions > Percent

Problems 17, 18, and 19 refer to the following:

Cookies For a Crowd

At Central Middle School the 108 students who take the AMC $\rightarrow$ 8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookies this year. Their recipe, which makes a pan of 15 cookies, list these items:
$1\frac{1}{2}$ cups of flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipe.



Walter can buy eggs by the half-dozen. How many half-dozens should be buy to make enough cookies? (Some eggs and some cookies may be left over.)

(A) 1 **(B)** 2 **(C)** 5 **(D)** 7 **(E)** 15

1999 AMC 8, Problem #17—
"There are a total of 216 cookies that will be consumed, each recipe makes 15 cookies. So $216 \div 15 \approx 15$ recipes are needed."

Solution

Answer (C): One recipe makes 15 cookies, so $216 \div 15 = 14.4$ recipes are needed, but this must be rounded up to 15 recipes to make enough cookies. Each recipe requires 2 eggs. So 30 eggs are needed. This is 5 half-dozens.

Difficulty: Medium
NCTM Standard: Algebra Standard for Grades 6-8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division

Problems 17, 18, and 19 refer to the following:

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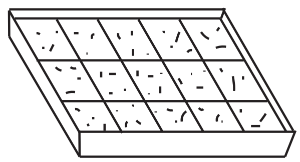

They learn that a big concert is scheduled for the same night and attendance will be down 25%. How many recipes of cookies should they make for their smaller party?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

1999 AMC 8, Problem #18—
"The number of Cookies that need to be prepared is $108(75\%)\times 2$."

Solution

Answer (E): The $108(0.75) = 81$ students need 2 cookies each so 162 cookies are to be baked. Since $162 \div 15 = 10.8$, Walter and Gretel must bake 11 recipes. A few leftovers are a good thing!

Difficulty: Medium-hard

NCTM Standard: Algebra Standard for Grades 6-8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division
Number Theory > Arithmetic > Fractions > Percent

Problems 17, 18, and 19 refer to the following:

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At Central Middle School the 108 students who take the AMC $\rightarrow$ 8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookies this year. Their recipe, which makes a pan of 15 cookies, list these items:
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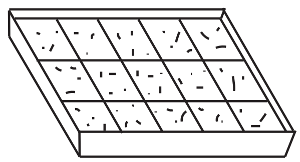


The drummer gets sick. The concert is cancelled. Walter and Gretel must make enough pans of cookies to supply 216 cookies. There are 8 tablespoons in a stick of butter. How many sticks of butter will be needed? (Some butter may be left over, of course.)

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

1999 AMC 8, Problem #19—
"They will have to bake 15 recipes of cookies."

Solution

Answer (B): Since $216 \div 15 = 14.4$, they will have to bake 15 recipes. This requires $15 \times 3 = 45$ tablespoons of butter. So, $45 \div 8 = 5.625$, and 6 sticks are needed.

Difficulty: Medium-hard

NCTM Standard: Algebra Standard for Grades 6-8: use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Multiplication and Division

Figure 1 is called a "stack map." The numbers tell how many cubes are stacked in each position. Fig. 2 shows these cubes, and Fig. 3 shows the view of the stacked cubes as seen from the front.
Which of the following is the front view for the stack map in Fig. 4?

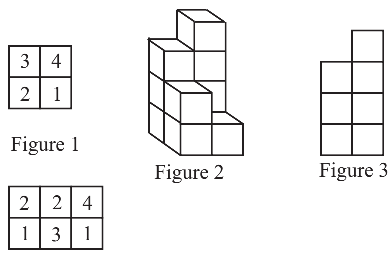
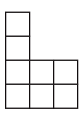
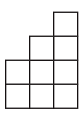
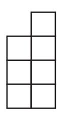
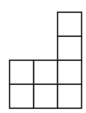
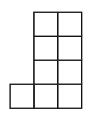


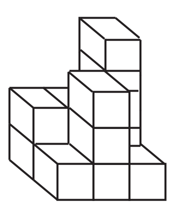
Figure 1 Figure 2 Figure 3
Figure 4

(A)  **(B)**  **(C)**  **(D)**  **(E)** 

1999 AMC 8, Problem #20—
"The front view shows the larger of the numbers of cubes in the front or back stack in each column."

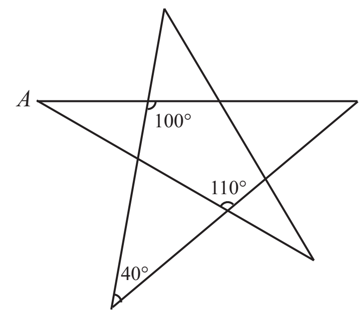
Solution

Answer (B): The front view shows the larger of the numbers of cubes in the front or back stack in each column. Therefore the desired front view will have, from left to right, 2, 3, and 4 cubes. This is choice B.



Difficulty: Medium-hard
NCTM Standard: Geometry Standard for Grades 6-8: analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
Mathworld.com Classification: Geometry > Solid Geometry > Polyhedra > Cubes
Geometry > Geometric Construction

The degree measure of angle A is

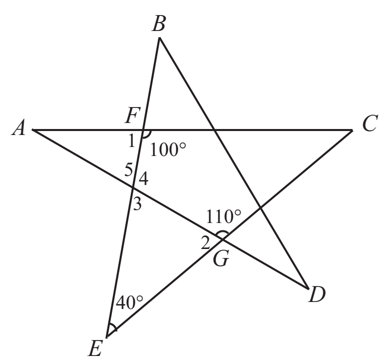


(A) 20 **(B)** 30 **(C)** 35 **(D)** 40 **(E)** 45

1999 AMC 8, Problem #21— "$\angle C = 180° - \angle E - \angle F$, and $\angle G = 110°$."

Solution

Answer (B): Since $\angle 1$ forms a straight line with angle $100°$, $\angle 1 = 80°$. Since $\angle 2$ forms a straight line with angle $110°$, $\angle 2 = 70°$. Angle 3 is the third angle in a triangle with $\angle E = 40°$ and $\angle 2 = 70°$, so $\angle 3 = 180° - 40° - 70° = 70°$.Angle $4 = 110°$ since it forms a straight angle with $\angle 3$. Then $\angle 5$ forms a straight angle with $\angle 4$, so $\angle 5 = 70°$. (Or $\angle 3 = \angle 5$ because they are vertical angles.) Therefore, $\angle A = 180° - \angle 1 - \angle 5 = 180° - 80° - 70° = 30°$.



OR

The angle sum in $\triangle CEF$ is $180°$, so $\angle C = 180° - 40° - 100° = 40°$. In $\triangle ACG$, $\angle G = 110°$ and $\angle C = 40°$, so $\angle A = 180° - 110° - 40° = 30°$.

Difficulty: Medium

NCTM Standard: Geometry Standard for Grades 6-8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Trigonometry > Angles
Geometry > Plane Geometry > Triangles > Special Triangles > Other Triangles > Triangle

In a far-off land three fish can be traded for two loaves of bread and a loaf of bread can be traded for four bags of rice. How many bags of rice is one fish worth?

(A) $\frac{3}{8}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{4}$ **(D)** $2\frac{2}{3}$ **(E)** $3\frac{1}{3}$

1999 AMC 8, Problem #22—
"One fish is worth $\frac{2}{3}$ of a loaf of bread, and each loaf of bread is worth four bags of rice."

Solution

Answer (D): One fish is worth $\frac{2}{3}$ of a loaf of bread and $\frac{2}{3}$ of a loaf of bread is worth $\frac{2}{3} \cdot 4 = \frac{8}{3} = 2\frac{2}{3}$ bags of rice.

OR

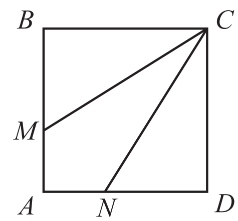
$$\begin{aligned} 3F &= 2B \\ \frac{3}{2}F &= B = 4R \\ (\frac{2}{3})(\frac{3}{2}) &= \frac{2}{3}(4R) \\ F &= \frac{8}{3}R = 2\frac{2}{3}R. \end{aligned}$$

Difficulty: Medium-hard

NCTM Standard: Number and Operations Standard for Grades 6-8: understand and use ratios and proportions to represent quantitative relationships.

Mathworld.com Classification: Number Theory > Arithmetic > Fractions

Square $ABCD$ has sides of length 3. Segments CM and CN divide the square's area into three equal part. How long is segment CM?

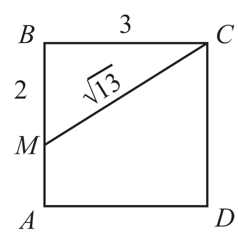


(A) $\sqrt{10}$ **(B)** $\sqrt{12}$ **(C)** $\sqrt{13}$ **(D)** $\sqrt{14}$ **(E)** $\sqrt{15}$

1999 AMC 8, Problem #23—
"Area of $\triangle MBC = (3 \times 3)\frac{1}{3} = \frac{1}{2}(MB)(BC)$."

Solution

Answer (C): One-third of the square's area is 3, so triangle MBC has area $3 = \frac{1}{2}(MB)(BC)$. Since side BC is 3, side MB must be 2. The hypotenuse CM of this right triangle is $\sqrt{2^2 + 3^2} = \sqrt{13}$.



Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 6-8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area

When 1999^{2000} is divided by 5, the remainder is

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1 **(E)** 0

1999 AMC 8, Problem #24— "Since any positive integer(expressed in base ten) is some multiple of 5 plus its last digit, its remainder when divided by 5 can be obtained by knowing its last digit."

Solution

Answer (D): Since any positive integer(expressed in base ten) is some multiple of 5 plus its last digit, its remainder when divided by 5 can be obtained by knowing its last digit.

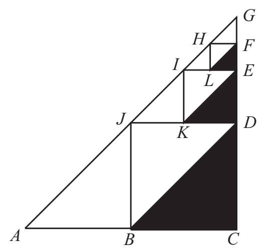
Note that 1999^1 ends in 9, 1999^2 ends in 1, 1999^3 ends in 9, 1999^4 ends in 1, and this alternation of 9 and 1 endings continues with all even powers ending in 1. Therefore, the remainder when 1999^{2000} is divided by 5 is 1 .

Difficulty: Medium-hard

NCTM Standard: Geometry Standard for Grades 6-8: understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.

Mathworld.com Classification: Calculus and Analysis > Special Functions > Powers
Number Theory > Arithmetic > Multiplication and Division > Remainder

Points B, D, and J are midpoints of the sides of right triangle ACG. Points K, E, I are midpoints of the sides of triangle JDG, etc. If the dividing and shading process is done 100 times(the first three are shown) and $AC = CG = 6$, then the total area of the shaded triangles is nearest



(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

1999 AMC 8, Problem #25—
"At each stage the area of the shaded triangle is one-third of the trapezoidal region not containing the smaller triangle being divided in the next step."

Solution

Answer (A): At each stage the area of the shaded triangle is one-third of the trapezoidal region not containing the smaller triangle being divided in the next step. Thus, the total area of the shaded triangles comes closer and closer to one-third of the area of the triangular region ACG and this is $\frac{1}{3} \cdot \frac{1}{2} \cdot 6 \cdot 6 = 6$. The shaded areas for the first six stages are: 4.5, 5.625, 5.906, 5.976, 5.994, and 5,668.

These are the calculations for the first three steps.

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} = 4.5$

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} + \frac{1}{2} \cdot \frac{6}{4} \cdot \frac{6}{4} = 4.5 + 1.125 = 5.625$

$\frac{1}{2} \cdot \frac{6}{2} \cdot \frac{6}{2} + \frac{1}{2} \cdot \frac{6}{4} \cdot \frac{6}{4} + \frac{1}{2} \cdot \frac{6}{8} \cdot \frac{6}{8} = 5.625 + 0.281 = 5.906$

Difficulty: Hard

NCTM Standard: Geometry Standard for Grades 6-8: understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Mathworld.com Classification: Geometry > Plane Geometry > Miscellaneous Plane Geometry > Area