Kangourou Sans Frontières

Mathematics Promotion Society *Math Kangaroo in USA*

Math Kangaroo 2012 in USA

International Competition in Mathematics
Thursday, March 15, 2012

Levels 7 and 8

This test consists of 30 questions on 4 pages.
You have 75 minutes to complete it.
Calculators are not allowed!
Please enter your answers on the answer form provided.
Please put your name and ID number on the line below.

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3 Point Problems

1. Four chocolate bars cost 6 dollars more than one chocolate bar. What is the cost of one chocolate bar?

A) 1 dollar B) 2 dollar C) 3 dollar D) 4 dollar E) 5 dollar

2. $11.11 - 1.111 =$

A) 9.009 B) 9.0909 C) 9.99 D) 9.999 E) 10

3. A watch is placed face up on a table so that its minute hand points northeast. How many minutes pass before the minute hand points northwest for the first time?

A) 45 B) 40 C) 30 D) 20 E) 15

4. Mary has a pair of scissors and five cardboard letters. She cuts each letter exactly once (along a straight line) so that it falls apart into as many pieces as possible. Which letter falls apart into the most pieces?

A) O B) F C) S D) H E) M

5. A dragon has five heads. Every time a head is chopped off, five new heads grow. If six heads are chopped off one by one, how many heads will the dragon have at the end?

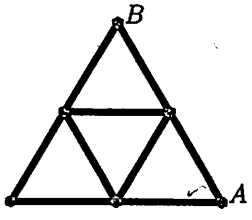
A) 25 B) 28 C) 29 D) 30 E) 35

6. In which of the following expressions can we replace each occurrence of the number 8 by the same positive number (other than 8) and obtain the same result?

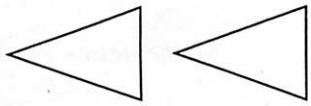
A) $(8+8)\div 8+8$ B) $8\times(8+8)\div 8$ C) $8+8-8+8$ D) $(8+8-8)\times 8$ E) $(8+8-8)\div 8$

7. Each of the nine paths in a park is 100 m long. Ann wants to go from A to B without going along any path more than once. What is the length of the longest route she can choose?

A) 900 m B) 800 m C) 700 m D) 600 m E) 400 m

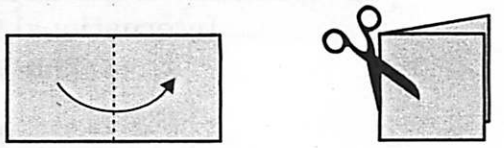


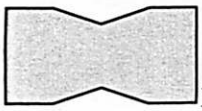
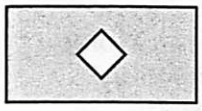
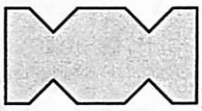
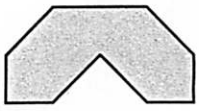
8. The diagram shows two triangles. In how many ways can you choose two vertices, one on each triangle, so that the straight line through the vertices does not cross either triangle?



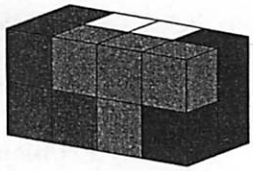
A) 1 B) 2 C) 3 D) 4 E) more than 4

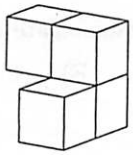
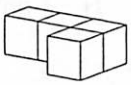
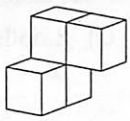
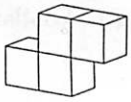
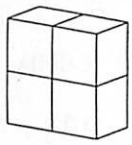
9. Werner folds a sheet of paper as shown in the figure and then makes two straight cuts with a pair of scissors. He then opens up the paper again. Which of the following shapes cannot be the result?



A)  B)  C)  D)  E) 

10. A rectangular prism is made up of four pieces, as shown. Each piece consists of four cubes and is a single color. What is the shape of the white piece?



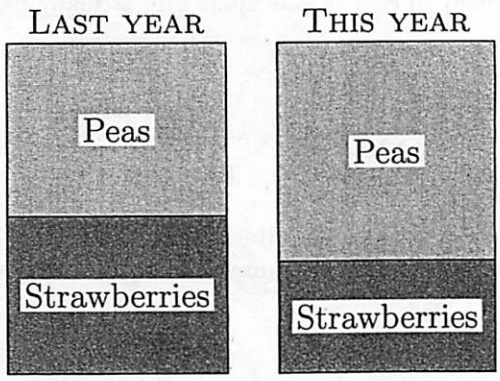
A)  B)  C)  D)  E) 

4 Point Problems

11. Kanga forms two 4-digit natural numbers using each of the digits 1, 2, 3, 4, 5, 6, 7 and 8 exactly once. Kanga wants the sum of the two numbers to be as small as possible. What is the value of this smallest possible sum?

A) 2468 B) 3333 C) 3825 D) 4734 E) 6912

12. Mrs. Gardner grows peas and strawberries. This year she has changed the rectangular pea bed to a square by lengthening one of its sides by 3 meters. As a result of this change, the area of the strawberry bed was reduced by 15 m^2. What was the area of the pea bed before the change?



A) 5 m^2 B) 9 m^2 C) 10 m^2 D) 15 m^2 E) 18 m^2

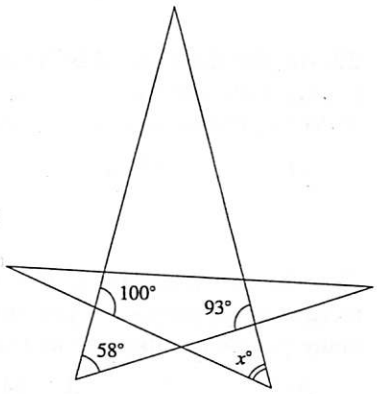
13. Barbara wants to complete the diagram by inserting three numbers, one in each empty cell. She wants the sum of the first three numbers to be 100, the sum of the three middle numbers to be 200 and the sum of the last three numbers to be 300. What number should Barbara insert in the middle cell of the diagram?

10				130

A) 50 B) 60 C) 70 D) 75 E) 100

14. In the figure to the right, what is the value of x?

A) 35 B) 42 C) 51 D) 65 E) 109

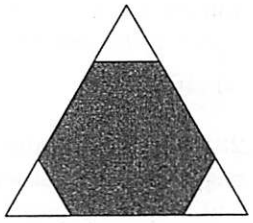


15. Four cards each have a number written on one side and a phrase written on the other. The four phrases are "divisible by 7," "prime," "odd" and "greater than 100," and the four numbers are 2, 5, 7 and 12. On each card, the number does not correspond to the phrase on the other side. What number is written on the same card as the phrase "greater than 100?"

A) 2 B) 5 C) 7 D) 12 E) It is impossible to determine.

16. Three small equilateral triangles of the same size are cut from the corners of a larger equilateral triangle with sides of 6 cm, as shown. The sum of the perimeters of the three small triangles is equal to the perimeter of the remaining gray hexagon. What is the side length of the small triangles?

A) 1 cm B) 1.2 cm C) 1.25 cm D) 1.5 cm E) 2 cm



17. A piece of cheese is cut into a large number of pieces. During the course of the day, a number of mice came and stole some pieces, watched by the lazy cat Ginger. Ginger noticed that each mouse stole a different number of pieces, each of which was less than 10, and that no mouse stole exactly twice as many pieces as any other mouse. What is the largest number of mice that Ginger could have seen stealing cheese?

A) 4 B) 5 C) 6 D) 7 E) 8

18. At the airport there is a moving walkway 500 metres long, which moves at a speed of 4 km/hour. Ann and Bill step on the walkway at the same time. Ann walks at a speed of 6 km/hour on the walkway while Bill stands still. When Ann comes to the end of the walkway, how far ahead of Bill is she?

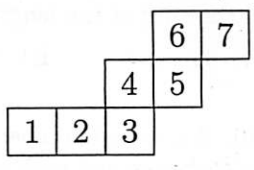
A) 100 m B) 160 m C) 200 m D) 250 m E) 300 m

19. A magical talking square originally has sides of length 8 cm. If he tells the truth, then his sides become 2 cm shorter. If he lies, then his perimeter doubles. He makes four statements, two true and two false, in some order. What is the largest possible perimeter of the square after the four statements?

A) 28 B) 80 C) 88 D) 112 E) 120

20. A cube is rolled on a plane so that it turns around its edges. It begins at position 1, and is rolled so that one of its faces touches the plane in positions 2, 3, 4, 5, 6, and 7, in that order, as shown. Which two of these positions were occupied by the same face of the cube?

A) 1 and 7 B) 1 and 6 C) 1 and 5 D) 2 and 7 E) 2 and 6

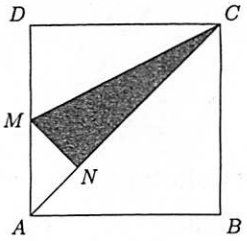


5 Point Problems

21. Rick has five cubes. When he arranges them from smallest to largest, the difference between the heights of any two neighboring cubes is 2 cm. The largest cube is as high as a tower built from the two smallest cubes. How high is a tower built from all five cubes?

A) 6 cm B) 14 cm C) 22 cm D) 44 cm E) 50 cm

22. In the diagram, $ABCD$ is a square, M is the midpoint of AD and MN is perpendicular to AC. What is the ratio of the area of the shaded triangle MNC to the area of the square?



A) 1:6 B) 1:5 C) 7:36 D) 3:16 E) 7:40

23. The tango is danced in pairs, each consisting of one man and one woman. At a dance evening no more than 50 people are present. At one moment 3/4 of the men are dancing with 4/5 of the women. How many people are dancing at that moment?

A) 20 B) 24 C) 30 D) 32 E) 46

24. David wants to arrange the twelve numbers from 1 to 12 in a circle so that any two neighboring numbers differ by either 2 or 3. Which of the following pairs of numbers have to be neighbors?

A) 5 and 8 B) 3 and 5 C) 7 and 9 D) 6 and 8 E) 4 and 6

25. Some three-digit integers have the following property: if you remove the first digit of the number, you get a perfect square; if instead you remove the last digit of the number, you also get a perfect square. What is the sum of all the three-digit integers with this curious property?

A) 1013 B) 1177 C) 1465 D) 1993 E) 2016

26. A book contains 30 stories, each starting on a new page. The lengths of the stories are 1, 2, 3, ..., 30 pages. The first story starts on the first page. What is the largest number of stories that can start on an odd-numbered page?

A) 15 B) 18 C) 20 D) 21 E) 23

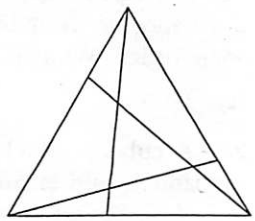
27. An equilateral triangle starts in a given position and is rotated into new positions in a sequence of steps. At each step it is rotated about its center, first by 3°, then by a further 9°, then by a further 27°, and so on (at the n-th step it is rotated by a further $(3^n)°$). How many different positions, including the initial position, will the triangle occupy? (Two positions are considered equal if the triangle covers the same part of the plane.)

A) 3 B) 4 C) 5 D) 6 E) 360

28. A rope is folded in half, then in half again, and then in half again. Finally the folded rope is cut through, forming several strands. The lengths of two of the strands are 4 m and 9 m. Which of the following could not have been the length of the whole rope?

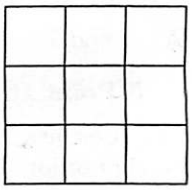
A) 52 m B) 68 m C) 72 m D) 88 m E) All the previous are possible.

29. A triangle is divided into four triangles and three quadrilaterals by three straight line segments (see the figure). The sum of the perimeters of the three quadrilaterals is equal to 25 cm. The sum of the perimeters of the four triangles is equal to 20 cm. The perimeter of the whole triangle is equal to 19 cm. What is the sum of the lengths of the three straight line segments?



A) 11 B) 12 C) 13 D) 15 E) 16

30. A positive number needs to be placed in each cell of the 3×3 grid shown, so that in each row and each column the product of the three numbers is equal to 1, and in each 2×2 square the product of the four numbers is equal to 2. What number should be placed in the central cell?



A) 16 B) 8 C) 4 D) $\frac{1}{4}$ E) $\frac{1}{8}$