1. C) 9

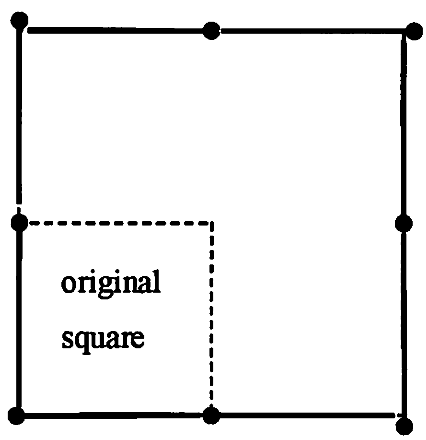
There are 9 different letters: VV, I, AAA, T, K, N, G, R, OO, so 9 colors are needed.

2. C) 1.5

$(6 - 3)/2 = 1.5$ m.

3. A) 8

A square containing 16 coins must have the area 4 times the square with 4 coins, so we have to double the size since $\sqrt{4} = 2$. Such a square needs 8 matches.



4. C) 142

There are 23 (skip #13 and #15) rows with 6 seats each and one row with 4 seats, so the total number of passenger seats is $23 \ast 6 + 4 = 142$.

5. E) 6 o'clock this morning

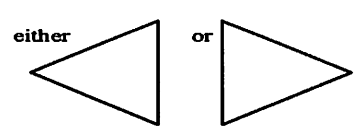
Now it is 5 o'clock in the afternoon in Madrid, so 17 hours after midnight in Madrid.

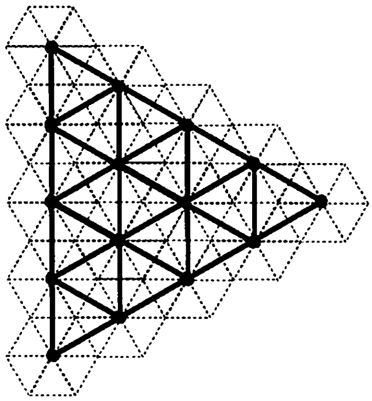
It is 8 o'clock in the morning on the same day in San Francisco, so 8 hours after midnight in San Francisco.

Ann went to bed in San Francisco at 9 o'clock yesterday evening, 3 hours before the midnight, so she went to bed $3 + 8 = 11$ hours ago.

In Madrid, 11 hours ago was $17 - 11 = 6$ hours after midnight in Madrid, so 6 o'clock this morning.

6. C) For any 3 neighboring hexagons the centers are vertices of an equilateral triangle with one side being vertical, so it is





When we move along the outer hexagons and connect the centers of neighboring hexagons, we form a bigger equilateral triangle and all small triangles (their shapes shown above) are inside the big equilateral triangle with the vertical side being to left, and there are 16 small triangles inside, so (C) is the answer.
Look at the figure to the left. Bold dots are centers of the hexagons and bold lines are connecting centers of neighboring hexagons. For convenience, each hexagon is shown as 6 equilateral triangles around its center.

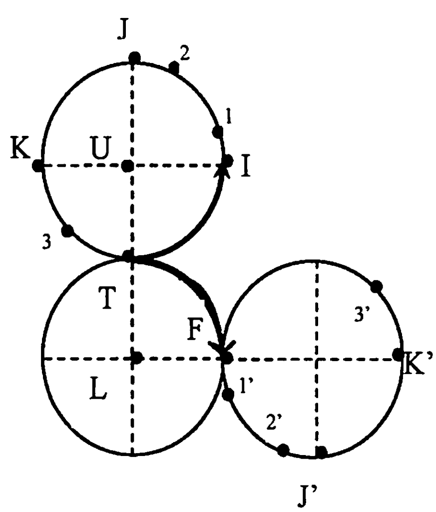
7. D) $(6 + 3)*2 + 1$

First step: $6 + 3$;

Second step: the result, $(6 + 3)$, is multiplied by 2, so we get $(6 + 3)*2$;

Third step: add 1 to the second step.

8. A)



Let U be the center of the upper coin, L the center of the lower coin, and let T be the initial tangent point. I is the point on the upper coin such that $\angle TUI$ is 90° counterclockwise, and F is the point on the lower coin such that $\angle TLF$ is 90° clockwise.
If we move the vertical line UL to the right it intersects at any moment some point P on the arc from T to I and the corresponding point P' on the arc from T to F. The arcs TP and TP' have the same length, so when the upper coin revolves around the lower coin, P touches the lower coin exactly at the point P'. In particular, I touches the lower coin at F, and so I' is F. But I corresponds to the kangaroo's forearms, so A) is the correct answer.

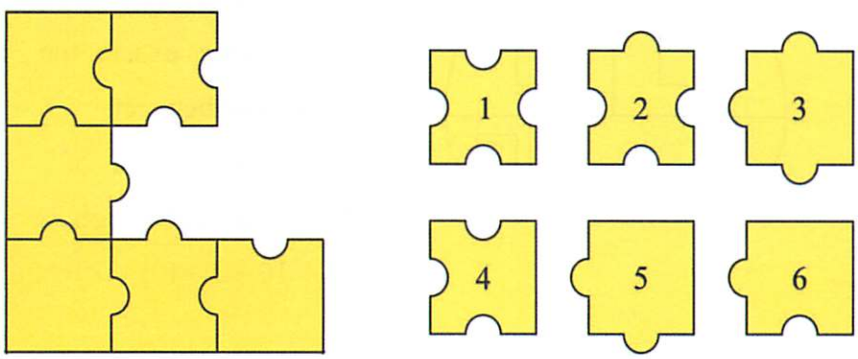
9.B) 20 kg

Adding one balloon allows you to lift an additional 180 kg – 80 kg = 100 kg. Since one balloon can lift items weighing 80 kg plus the basket, the basket must weigh 20 kg.

10. B) 13

They ate 9 pieces of fruit (1 apple + 3 pears + 3 apples + 2 pears), so they brought home 16 pieces of fruit. One half of the 16 pieces are pears, so the total number of pears they got from their grandmother was $8 + 3 + 2 = 13$.

11. D) 2, 3, 6



The piece #2 is the only one that fits to the center of the square. Move it first. Then place #3 to the right of #2 (since placing #5 will not allow the puzzle to be finished), and finally #6 on the top of #3.

12. B) B

The die we can't see is adjacent (on the top) to a C-die. On the bottom level the die is adjacent to an A-die (look at the left lower corner) and to a D-die (look at the right lower corner), so the die we can't see must be a B-die.

13. D) 3

For five cities, call them A, B, C, D, and E, there are the10 connections listed below:

{A,B}, {A,C},{A,D},{A,E},{B,C},{B,D},{B,E},{C,D},{C,E},and {D,E}.

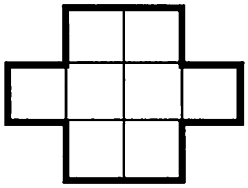
Seven of them are visible, so 3 are invisible.

14. C) only green

Any number of the form	1 + a multiple of 3	is red,
any number of the form	2 + a multiple of 3	is blue,
and any number of the form	3 + a multiple of 3	is green.

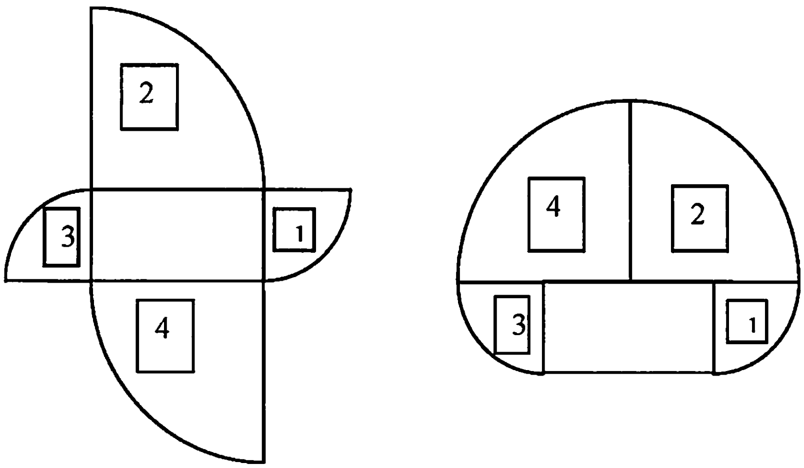
The sum of a red number and a blue number is 1+a multiple of 3 + 2+a multiple of 3, so it has the form 3 + a multiple of 3 and must be green.

15. D) 72 cm^2



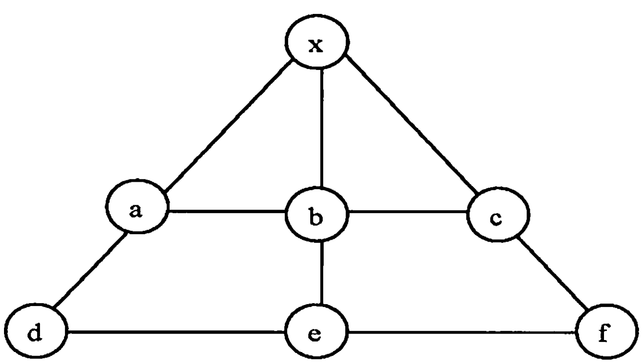
The figure consists of 8 identical squares of the size $s \times s$.
The perimeter of the figure is equal to 14s, so $s = \frac{42}{14} = 3$ and
the area of the figure $= 8s^2 = 8*3^2 = 72$ cm^2.

16. D) 20 cm



The curved parts of both perimeters are identical, so the difference between perimeters is exactly the difference between LINEAR segments of the perimeters, which is $(5 + 10 + 5 + 10) - 10 = 20$.

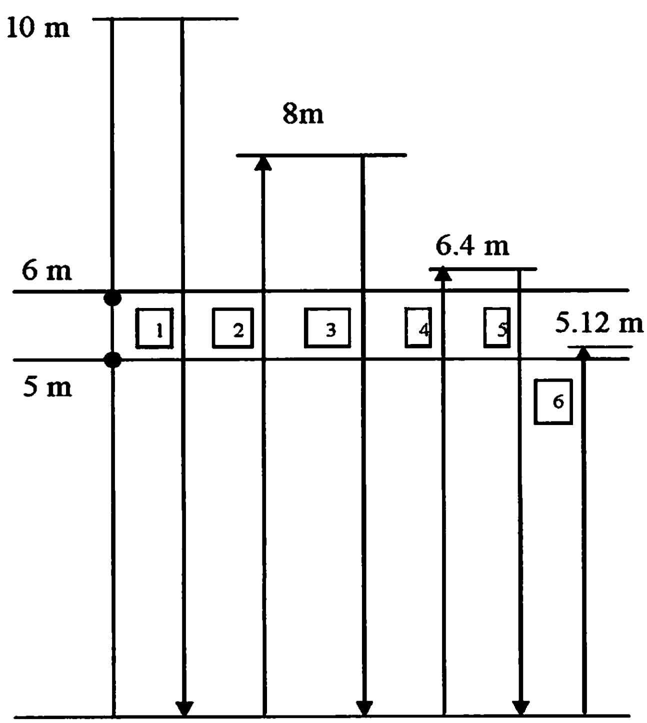
17. C) 4



There are five lines and along each line the sum of the three numbers is equal to a fixed number S.
Let T be the sum of all seven numbers.
The two horizontal lines contain all numbers except the vertex x, so
$2S = T - x$.
The three lines passing through the vertex contain all seven numbers, with x included two additional times, so $3S = T + 2x$.
Multiply the 1st equation by 3 and the 2nd equation by 2.
Then $6S = 3T - 3x$ and $6S = 2T + 4x$. Next, $2T + 4x = 3T - 3x$, so $7x = T$.
Therefore, $x = T/7$ and $S = 3T/7$. In our exercise $T = 28$, so $x = 4$.

One of possible solutions is $a = 1$, $b = 5$, $c = 6$, $d = 7$, $e = 3$, $f = 2$, and $x = 4$.

18. D) 6



10, 8, 6.4, and 5.12 m are the highest levels the ball reaches **before** the 1st, 2nd, 3rd, 4th bounce, respectively. The next height is $5.12 * .8 = 4.096$ m, which is below the bottom of the window.

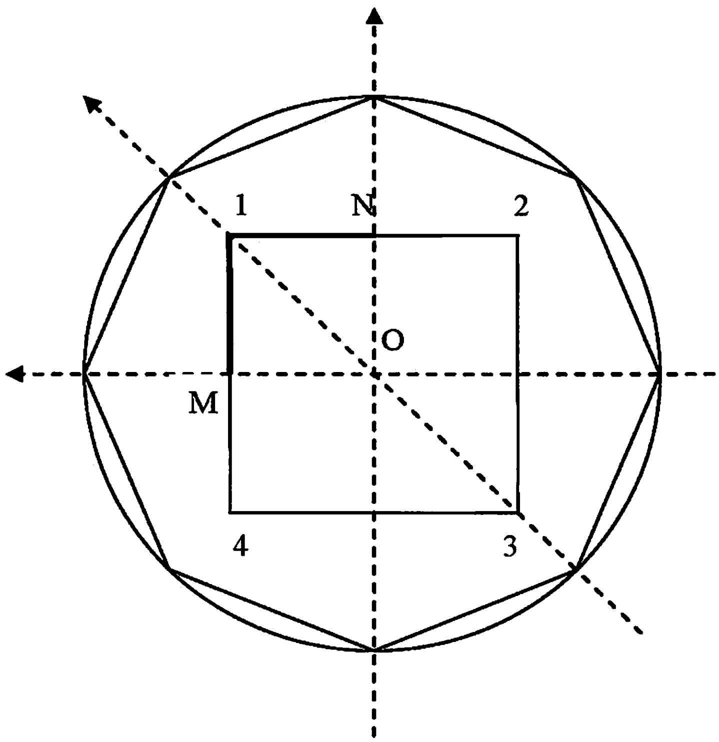
When the ball reaches 5.12 m, it moves down in front of the window, but once it drops below the window it never appears in front of it, so the answer is 6.

19. A) 3

Note that every time the first wheel moves one gear, one gear is moved on the second wheel, which in turn moves one gear on the third wheel, which finally moves one gear on the last wheel. So if the first wheel moves one revolution, it moves 30 gears, which means the last wheel moves 30 gears as well. Since there are only 10 gears on the last wheel, the last wheel must move a total of $30/10 = 3$ revolutions.

20. C)

A regular octagon inscribed in the circle with the center O is shown below.
The three axes shown are the folding lines. The vertical axis is the 1st folding line, the horizontal axis is the second folding line, and the diagonal axis is the 3rd folding line.
Inside the octagon is a square with the center at the intersection of folding lines and sides parallel to the vertical and horizontal axes, respectively.



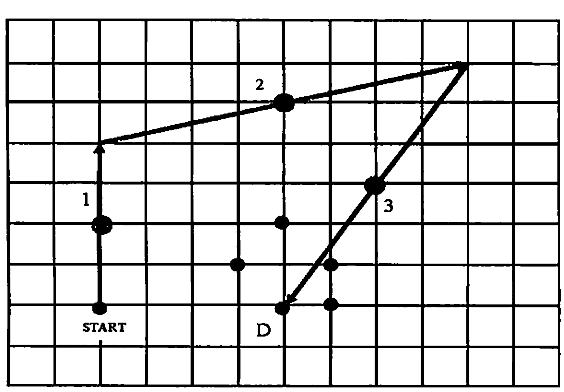
The vertices of the square are enumerated as shown, so that M is the midpoint of the side1-4, and N is the midpoint of the side 1-2.

After the 1st folding 2 goes to 1, 3 goes to 4, and the side 2-3 covers the side 1-4. After the 2nd folding all vertices go to 1 and the square folds into the square with the vertices 1, N, O, M. After the 3rd folding M goes to N, and O, 1, N are not moved at any of the three stages.

The whole square has folded to the triangle ΔO1 N. 1-N is the cutting line. The configuration shown above, a square inside the octagon with sides parallel to the first two folding lines (so <u>not</u> parallel to any side of the octagon), is the answer.

21. B) The amount of vinegar and water together is $\frac{5}{6}$ of the amount of wine
We know that the mount of vinegar = $\frac{1}{2}$ amount of wine, and the amount of water = $\frac{1}{3}$ amount of wine, so the amount of Vinegar + Water $= (\frac{1}{2} + \frac{1}{3}) = \frac{5}{6}$ of wine.

22. D) Point D



23. D) 7.5

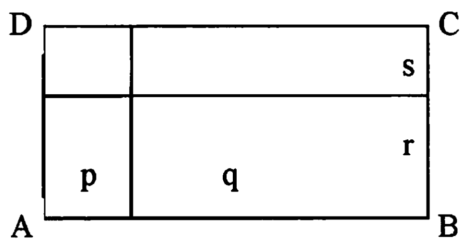
The smallest number of children of age different than 8 is

4 (6 years old) + 1 (7 years old) + 1 (9 years old) + 1 (10 years old) = 7.

The number of 8 years old children must be ≥ 5 (the most common age) to exceed the number of 6 years old children. 7 + 5 = 12 (all kids), so the number of 8 years old children must be exactly 5. Also, there is exactly one child for each age different than 6 and 8.

The average age = $\frac{4\times6+7+5\times8+9+10}{12}$ = 7.5.

24. B) 30



Let p + q = AB and r + s = BC. Suppose that p < q and s < r. The perimeters of the smaller rectangles are: 2(q+r) in the lower right, 2(p+r), 2(q+s), and 2(p+s) in the upper left.

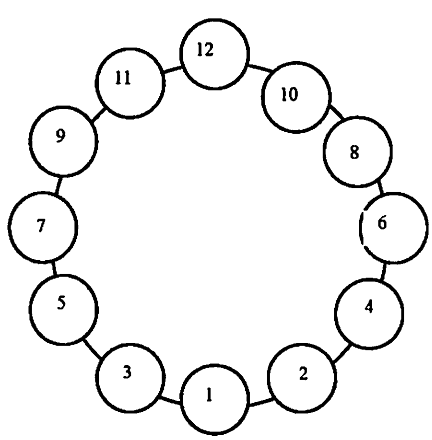
2(p+s) is the smallest perimeter and 2(q+r) is the biggest one.

Therefore, 2(p+s) = 11 and 2(q+r) = 19. 2(p+s) + 2(q+r) = 11 + 19 = 30.

2(p+s) + 2(q+r) = (p+q) + (r+s) + (q+p) + (s+r) = AB + BC + CD + DA which is the perimeter of the original rectangle ABCD.

Therefore, the perimeter of the original rectangle ABCD is 30.

25. D) 8 and 10



Any rotation of the circle or symmetry with respect to any diagonal of the circle does not change neighboring pairs (even if the order is changed), so we may assume that 12 is at the top of the circle and 11 is to the left (like in an ordinary clock). Then 10 is to the right of 12 since the only possible neighbors of 12 are 10 and 11 (there is no13 or 14).

Moving down from 12 in both directions:

(i) below 11 you only can have 9 since 10, 12 are not available, and then below 10 you only can have 8 since 9, 11, 12 are not available;

(ii) below 9 you only can have 7 since 8, 10,11 are not available and then below 8 you only can have 6 since 7, 9,10 are not available;

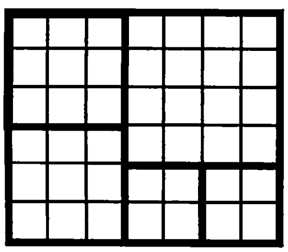
(iii) below 7 you only can have 5 since 6, 8, 9 are not available and then below 6 you only can have 4 since 5, 7, 8 are not available.

Repeat the argument to see that the above "clock" is the only solution (up to rotation/symmetry). Notice that the opposite numbers add up to 13.

From the list of given options only 8 and 10 are neighbors.

26. B) 5

There is a partition using 5 squares.



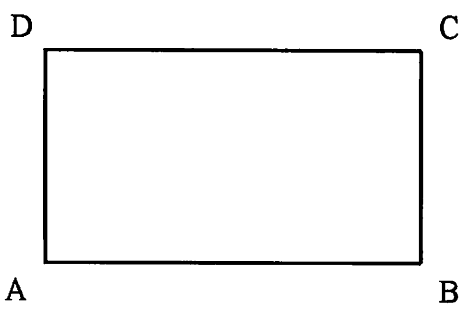
If we cut the rectangle into any number of squares, at each corner of the rectangle we must have a square fitting exactly into that corner. If two vertices belong to the same square, then its size is 6×6 (7×7 is too big), with one column of size 6x1. The area of any square inside that column can't exceed 1, so at least 6 squares are needed to cover the column; 1 + 6 is more than 5.

Let's analyze the other option; different corners belong to different squares.

If there is a part of the rectangle that is not covered by the 4 corner squares, we need the fifth square so 5 is still the minimal number.

Suppose that the 4 corner squares do not overlap and cover the whole rectangle.

That is impossible for any rectangle that is not a square.



To see this, let a×a, b×b, c×c, and d×d be the sizes of squares at the corners A,B, C, and D, respectively. Let w = the width of the rectangle and ℓ = the length of the rectangle.

$a + d = w = b + c$, so $a + b + c + d = 2w$.

$a + b = \ell = c + d$, so $a + b + c + d = 2\ell$.

Therefore,

$2w = 2\ell$, which can happen only for squares.

In conclusion, we need at least 5 squares to cover the 6×7 rectangle.

27. D)

R			R	2
		R		1
	R	R		2
R			R	2
2	1	2	2	

We can go one step further and show that any other option from the list is impossible. (B) is quite easy. Count the number of red cells row by row to get $1 + 2 + 1 + 3 = 7$.
Count the same cells column by column to get $2 + 2 + 3 + 1 = 8$.
Contradiction! $7 \neq 8$.

(A) indicates that the top row has only red cells which contradicts the fact that the leftmost column doesn't have red cells.

(C) indicates that the two bottom rows have no red cells but then we can't have more than 2 red cells in the second column which contradicts number 3 for this column.

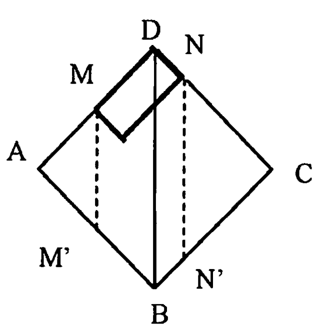
(E) defines the following 3×3 square (by removing the top row and the leftmost column).

			3
			3
			1
3	1	3	

The top 2 rows contain only red cells, so in each column we have at least 2 red cells which contradicts the number 1 for the middle column.

28. D) 16 cm^2

Let A, B, C, D be vertices of the square. Let 2h be the length of the diagonal BD. Then the distance from A to BD is h, so the first folding line is $\frac{h}{2}$ units from BD and $\frac{3h}{2}$ units from the vertex C. The second folding line is parallel to the first one and exactly in the middle between C and the first folding line. Hence, the distance from the first folding line to the diagonal BD is $\frac{h}{2}$ and from BD to the second folding line the distance is $\frac{h}{4}$.



The rectangle with the bold perimeter and vertices M, D, N is one-half of the shaded area in this exercise. According to the distances computed above and the corresponding proportions, DM $= \frac{1}{2}s$ and DN $= \frac{1}{4}s$, where s is the side of the square. The area of the rectangle = DM$*$DN $= \frac{1}{8}s^2$, so the shaded area $= \frac{1}{4}s^2 = \frac{1}{4} * 64 = 16$ cm^2.

29. C) 5

Let abc be Abid's house number. Then bc is Ben's house number and c is Chiara's house number. A house number never begins with the digit 0, so $1 \le a, b, c \le 9$ and the sum of house numbers, $(100a + 10b + c) + (10b + c) + c$, is 912.

Thus, $100a + 20b + 3c = 912$. $20(5a + b) = 3(304 - c)$, so $304 - c$ is a multiple of 20.

$c = 4$ is the only option since $1 \le c \le 9$. Hence, $20(5a + b) = 900$ and $5a + b = 45$.

$b = 5(9 - a)$, so b is a multiple of 5. From 1 to 9 only 5 is a multiple of 5. Therefore, $b = 5$.

$a = (912 - 20*5 - 3*4)/100 = 8$ and Abid's house number is 854.

30. B) 3

Anybody who gets 1 knows that the other person got 2.

Ann doesn't have 1 and Bill gains this piece of information from Ann's first statement.

Bill doesn't have 1. Otherwise he would know Ann's number.

Bill doesn't have 2. If he had 2, he would know Ann's number as 3 since 1 is already excluded, so he does not have 2 and Ann knows it.

So, Ann must have 3 to know Bill's number at this moment.

His number is 4 (2 is already excluded) and it is a divisor of 20.