

AMERICAN MATHEMATICS COMPETITIONS

16TH ANNUAL



TUESDAY, NOVEMBER 14, 2000

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score on this test is the number of correct answers.
5. No aids are permitted other than scratch paper, graph paper, rulers, erasers, and calculators that are accepted for use on the SAT. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have 40 minutes to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

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1. Aunt Anna is 42 years old. Caitlin is 5 years younger than Brianna, and Brianna is half as old as Aunt Anna. How old is Caitlin?

(A) 15 (B) 16 (C) 17 (D) 21 (E) 37

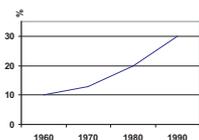
2. Which of these numbers is less than its reciprocal?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

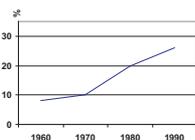
3. How many whole numbers lie in the interval between $\frac{5}{3}$ and 2π ?

(A) 2 (B) 3 (C) 4 (D) 5 (E) infinitely many

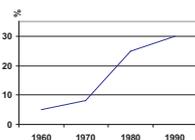
4. In 1960 only 5% of the working adults in Carlin City worked at home. By 1970 the “at-home” work force had increased to 8%. In 1980 there were approximately 15% working at home, and in 1990 there were 30%. The graph that best illustrates this is:



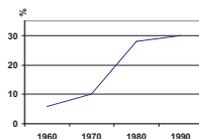
(A)



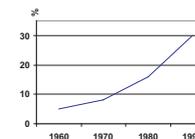
(B)



(C)



(D)



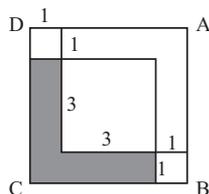
(E)

5. Each principal of Lincoln High School serves exactly one 3-year term. What is the maximum number of principals this school could have during an 8-year period?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 8

6. Figure $ABCD$ is a square. Inside this square three smaller squares are drawn with side lengths as labeled. the area of the shaded L-shaped region is

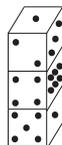
(A) 7 (B) 10 (C) 12.5 (D) 14 (E) 15



7. What is the minimum possible product of three different numbers of the set $\{-8, -6, -4, 0, 3, 5, 7\}$?

(A) -336 (B) -280 (C) -210 (D) -192 (E) 0

8. Three dice with faces numbered 1 through 6 are stacked as shown. Seven of the eighteen faces are visible, leaving eleven faces hidden (back, bottom, between). The total number of dots NOT visible in this view is



(A) 21 (B) 22 (C) 31 (D) 41 (E) 53

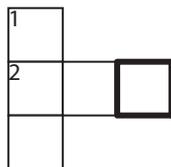
9. Three-digit powers of 2 and 5 are used in this *cross-number* puzzle. What is the only possible digit for the outlined square?

ACROSS

2. 2^m

DOWN

1. 5^n



(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

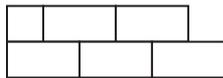
10. Ara and Shea were once the same height. Since then Shea has grown 20% while Ara has grown half as many inches as Shea. Shea is now 60 inches tall. How tall, in inches, is Ara now?

(A) 48 (B) 51 (C) 52 (D) 54 (E) 55

11. The number 64 has the property that it is divisible by its units digit. How many whole numbers between 10 and 50 have this property?

(A) 15 (B) 16 (C) 17 (D) 18 (E) 20

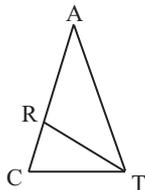
12. A block wall 100 feet long and 7 feet high will be constructed using blocks that are 1 foot high and either 2 feet long or 1 foot long (no blocks may be cut). The vertical joins in the blocks must be staggered as shown, and the wall must be even on the ends. What is the smallest number of blocks needed to build this wall?



- (A) 344 (B) 347 (C) 350 (D) 353 (E) 356

13. In triangle CAT , we have $\angle ACT = \angle ATC$ and $\angle CAT = 36^\circ$. If \overline{TR} bisects $\angle ATC$, then $\angle CRT =$

- (A) 16° (B) 51° (C) 72° (D) 90° (E) 108°

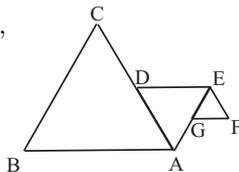


14. What is the units digit of $19^{19} + 99^{99}$?

- (A) 0 (B) 1 (C) 2 (D) 8 (E) 9

15. Triangle ABC , ADE , and EFG are all equilateral. Points D and G are midpoints of \overline{AC} and \overline{AE} , respectively. If $AB = 4$, what is the perimeter of figure $ABCDEFG$?

- (A) 12 (B) 13 (C) 15 (D) 18 (E) 21



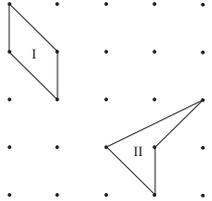
16. In order for Mateen to walk a kilometer(1000m) in his rectangular backyard, he must walk the length 25 times or walk its perimeter 10 times. What is the area of Mateen's backyard in square meters?

- (A) 40 (B) 200 (C) 400 (D) 500 (E) 1000

17. The operation \otimes is defined for all nonzero numbers by $a \otimes b = \frac{a^2}{b}$. Determine $[(1 \otimes 2) \otimes 3] - [1 \otimes (2 \otimes 3)]$.

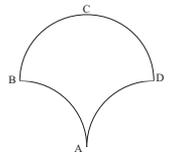
(A) $-\frac{2}{3}$ (B) $-\frac{1}{4}$ (C) 0 (D) $\frac{1}{4}$ (E) $\frac{2}{3}$

18. Consider these two geoboard quadrilaterals. Which of the following statements is true?



- (A) The area of quadrilateral I is more than the area of quadrilateral II.
- (B) The area of quadrilateral I is less than the area of quadrilateral II.
- (C) The quadrilaterals have the same area and the same perimeter.
- (D) The quadrilaterals have the same area, but the perimeter of I is more than the perimeter of II.
- (E) The quadrilaterals have the same area, but the perimeter of I is less than the perimeter of II.

19. Three circular arcs of radius 5 units bound the region shown. Arcs AB and AD are quarter-circles, and arc BCD is a semi-circle. What is the area, in square units, of the region?



- (A) 25 (B) $10 + 5\pi$ (C) 50 (D) $50 + 5\pi$
- (E) 25π

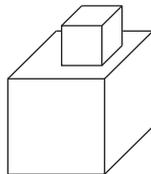
20. You have nine coins: a collection of pennies, nickels, dimes, and quarters having a total value of \$1.02, with at least one coin of each type. How many dimes must you have?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

21. Keiko tosses one penny and Ephraim tosses two pennies. The probability that Ephraim gets the same number of heads that Keiko gets is

(A) $\frac{1}{4}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

22. A cube has edge length 2. Suppose that we glue a cube of edge length 1 on top of the big cube so that one of its faces rests entirely on the top face of the larger cube. The percent increase in the surface area (sides, top, and bottom) from the original cube to the new solid formed is closest to:



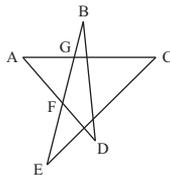
(A) 10 (B) 15 (C) 17 (D) 21 (E) 25

23. There is a list of seven numbers. The average of the first four numbers is 5, and the average of the last four numbers is 8. If the average of all seven numbers is $6\frac{4}{7}$, then the number common to both sets of four numbers is

(A) $5\frac{3}{7}$ (B) 6 (C) $6\frac{4}{7}$ (D) 7 (E) $7\frac{3}{7}$

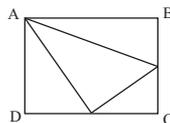
24. If $\angle A = 20^\circ$ and $\angle AFG = \angle AGF$, Then $\angle B + \angle D =$

(A) 48° (B) 60° (C) 72° (D) 80° (E) 90°



25. The area of rectangle $ABCD$ is 72. If point A and the midpoints of \overline{BC} and \overline{CD} are joined to form a triangle, the area of that triangle is

(A) 21 (B) 27 (C) 30 (D) 36 (E) 40



SOLUTIONS

Your School Manager will be sent at least one copy of the 2000 AMC 8 Solutions Pamphlet. It is meant to be loaned to students (but not duplicated).

WRITE TO US

Comments about the problems and solutions for this AMC 8 should be addressed to:

Prof. Joseph W. Kennedy, AMC 8 Chair / kennedj@muohio.edu
Department of Mathematics and Statistics, Miami University, Oxford, OH 45056

Comments about administrative arrangements should be addressed to:

Prof. Titu Andreescu, AMC Director / titu@amc.unl.edu
American Mathematics Competitions, University of Nebraska-Lincoln
P.O. Box 81606, Lincoln, NE 68501-1606

AMC 10 & AMC 12

The AMC 10 and AMC 12 are 25-question, 75-minute contests with 5 choices of answers for each problem (A through E). Schools with high scoring students on the AMC 8 will receive an Invitation Brochure for the 2000 AMC 10. The best way to prepare for these upper level contests is to study exams from previous years. Orders for all publications listed below should be addressed to:

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P.O. Box 81606
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PUBLICATIONS

MINIMUM ORDER: \$10 (Before handling fee), *PAYMENTS IN US FUNDS ONLY*. U.S.A. and Canadian orders must be prepaid and will be shipped UPS or AIRMAIL.

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Make checks payable to the American Mathematics Competitions; or give your Visa, MasterCard or American Express number, expiration date and cardholder's home address.

International Orders: Do NOT prepay; an invoice will be sent to you.

Each price is for an exam and its solutions for one year. Specify the years you want and how many copies of each exam. All prices effective to September 1 2001.

AMC 8 (Junior High/Middle School exam), 1985-2000, \$1.00 per copy per year.

AMC 10 & AMC 12 (High School Exam), 1990-2000, \$1.00 per copy per year.

Books (Exams and Solutions)

Problem Book I, AHSMEs 1950-1960,	\$10.00
Problem Book II, AHSMEs 1961-1965,	\$10.00
Problem Book III, AHSMEs 1966-1972,	\$13.00
Problem Book IV, AHSMEs 1973-1982,	\$13.00
Problem Book V, AHSMEs & AIMEs 1983-1988,	\$30.00
Problem Book VI, AHSMEs 1989-1994	\$24.00

2000

AMC 8

**DO NOT OPEN UNTIL
TUESDAY, NOVEMBER 14, 2000**

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE NOVEMBER 14, 2000.** Nothing is needed from inside this package until November 14.
2. Your **PRINCIPAL** or **VICE-PRINCIPAL** must verify on the **AMC 8 CERTIFICATION** Form that all rules associated with the conduct of the exam were followed.
3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.
4. **THE AMC 8 IS TO BE ADMINISTERED DURING A CONVENIENT 40 MINUTE PERIOD. THE EXAM MAY BE GIVEN DURING A REGULAR MATH CLASS.**
5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the copyright law.*

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